# Learning to Commit

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I study learning in auctions under limited commitment. In each period, the seller sets the terms for an auction selling an indivisible good among multiple buyers; but if the item fails to sell, he cannot pre-commit to the terms of future offerings. I find that, in interdependent value settings, the seller's equilibrium revenues are greater than immediately running an efficient, Vickrey auction. In contrast with private value settings, this result persists regardless of how often agents interact. This is because learning among buyers ensures lowers buyer valuations and ensure that the seller stops re-offering his good in finite time.

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Coase (1972) illustrated how limited commitment (i.e., sequential rationality) reduces profits. A monopolists sequentially offers a durable good to patient buyers. In each period, high valuation buyers are more likely to purchase the good than their low valuation peers. In response, the seller progressively lowers prices, which delays purchases and lowers profits. Such intuition generalizes to auctions (Liu et al 2019 and Vincent and McAfee 1997) and is a key consideration in dynamic contracting when the principal has limited commitment (Skreta 2006,2015 and Doval and Skreta 2022).

The argument above, however, depends on buyers knowing their valuation. This assumption seldom holds. For example, buyers participating in an art auction differ in their taste for art, but all value a painting's resale value. Since an artwork's resale value is often unknwon and buyers

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are differently informed of its realization, bidding behavior informs valuations. Does learning among buyers consequently changes the logic posited by Coase in a significant manner? I find that learning among buyer (at least partially) contravenes the Coase conjecture in the case of auctions.

As previously mentioned, art auctions are an ideal test-case for the dynamics described by Coase i.e., Coasian dynamics. Do art auctions provide empirical evidence for Coasian dynamics? Using an Impressionist art auction data (in appendix D), I analyze how often artworks that fail to sell at auction and how frequently are those artworks re-auctioned. I find that 30 percent of paintings brought up to auction failed to sell, but that 90 percent of these artworks were re-auctioned at a later date. Furthermore, even when artworks that failed to sell at auction are re-auctioned, sellers wait (on average) more than 5 years and pick a different auction house and location more than 65 percent of the time. This implies most real-world sellers avoid re-auctioning artwork failing to sell in spite of being sequentially rational. I claim that interdependence in valuations (by itself) can rationalize this behavior.

In this paper, I study auctions under limited commitment when buyers have interdependent valuations. In each period, the seller runs a second-price auction with a reserve price; but if the item fails to sell, he cannot pre-commit to future reserve prices. I find that as the item remains unsold, buyers become increasingly pessimistic of their peers' private information and subsequently lower their willingness to pay. After finitely many periods, the seller expects to value the good more than buyers and keeps his good from henceforth. My main result states that learning dynamics (as described below) ensures a revenue floor that does not depend on how often the seller transacts with buyers.

Under mild conditions, it is further the case that the seller's revenues equal the maximum revenues attained with full commitment i.e., when the seller is not required to be sequentially rational. The appendix further clarifies that the paper's insights persist in a broad class of auction and non-auction environments. In the following paragraph, I described in more detail how learning among buyers precludes serial auctions but delegate a formal example to section I.

Intuition Behind Main Results. — The results are clear when an item has a common value. Section I presents this example in detail, but I summarize the intuition below. Each buyer first receives private signals regarding the item's common value. Then, the seller sequentially auctions his item until it sells. If the seller has full commitment, he immediately runs his revenue maximizing (optimal), static auction and never re-auctions his good. Otherwise, he has limited commitment and may prefer to re-auction his good if it failed to sell and this reduces profits. Still, I find conditions for when a seller with limited commitment avoids re-auctioning his good.

The seller immediately runs the optimal, static auction. If the item sells, the game ends, so there is no commitment issue. Otherwise, each buyer expects that his peers are more likely to wait when they observe bad rather than good news regarding the common value. This prompts buyers to reduce their valuation for the good. In turn, this fall in valuations reduces the probability that a buyer values the good more than the seller. When private signals have bounded precision and there are enough buyers, learning among buyers expunges the gains from trade. Thus, the seller opts to keep his good.

In general, learning ensures that the seller stops re-auctioning his good in finite time and this imposes a revenue floor. This is driven by two effects. First, there always exist auctions where upon the good failing to sell, learning ensures that there are no additional gains from trade. The second effect is that learning further enables the seller to extract an increasing share of the remaining trade surplus. This is because learning among buyers lowers the dispersion in valuations over time, which increases competition among buyers.

The effects discussed above are implications of a technical result called progressive pessimism. This result states that in every period, buyers' initial beliefs regarding their peers' willingness to pay likelihood ratio dominate their posterior beliefs. Informally, this means that buyers beliefs regarding their peer's private information is stochastically decreasing.

In section IV, I present an additional implication of progressive pessimism: it ameliorates the winner's curse over time. Buyers bid a fraction of their expected valuation because the winning buyer expects to have received an overly optimistic estimate of his valuation. However, buyers screen their peers over time and increasingly expect to be able to win the auction, regardless of the common value. This implies that winning the good is increasingly uninformative and hence competition among buyers ensures that buyers to bid an increasing share of their current valuation.

The paper is organized at follows. Section I presents the main results in a stylized model. I then introduce the general model in section II and the main results are in section III. Section IV then presents the winner's curse results. Next, section V discusses the related literature. Lastly, section VI presents the discussion and concludes.

## I. Motivation: Selling Art.

This section illustrates the paper's main contribution, i.e. learning among buyers can contravene the Coase conjecture. A seller can auction a painting to a fixed set of patient buyers until the artwork sells. If the seller has full commitment, he is better off immediately running the revenue maximizing, static auction and keeping the painting if it fails to sell. Keeping paintings failing to sell, however, is seldom sequentially rational.

When buyer valuations are independent and the seller has limited commitment (i.e., he is sequentially rational), Liu et al (2019) proves that the seller's revenues converge to immediately running an efficient auction as the seller transacts with buyers increasingly frequently. This is the auction analog of the Coase conjecture. Meanwhile, when buyer valuations are interdependent, I show that the seller can attain strictly higher revenues, regardless of how often the seller transacts with his buyers. In fact, I find conditions for when the seller can immediately run the revenue maximizing, static auction and find it sequentially rational to keep his item when it fails to sell.

### *A. Environment*

A seller offers a painting to $n \geq 2$ buyers. Each buyer $i$ has a valuation $v_i$ that depends multiplicatively on a private value $\theta_i$ and a common value $q$ i.e.,

$$v_i \equiv \theta_i q.$$

The private values ($\theta_i$) are drawn iid uniformly between 0 and 1; meanwhile, $q$ is high (i.e., $q = 1$) with probability $\lambda \in (0, 1)$ or otherwise low ($q = 0$). I assume that $q$ is independent of the private values. Next, each buyer $i$ further observes a signal $x_i$ that is good news (i.e., $x_i = 1$) or bad news ($x_i = 0$). Conditional on $q$, I assume that the signals are drawn iid, but for each buyer $i$ it holds that $x_i = q$ with probability $\pi \in (1/2, 1)$. This assumption implies that the signals are informative and that the expected common value conditional on observing a good signal is higher than observing a bad signal: $E[q|x_i = 1] > E[q|x_i = 0]$.

Meanwhile, the seller has a commonly known valuation, called $\theta_s$, that does not depend on the common value $q$. This assumption is important since it simplifies exposition and avoids the issues associated with an informed seller i.e., the seller does not observe a private signal that would inform buyer valuations. Also, the results presented below can be extended to the case when the seller's valuation depends on $q$: seller's valuation is $\theta_s(q)$. I further assume, for exposition, that the seller's valuation is higher than buyers observing bad news but smaller than some buyers observing good news i.e.,

$$E[q|x_i = 0] < \theta_s < E[q|x_i = 1].$$

Next, the timing of play is as follows. Nature first draws the common value $q$, private values $(\theta_i)$, and the signals $(x_i)$. It then privately informs each buyer $i$ of $(\theta_i, x_i)$. In period $t = 0, 1, \ldots$, the seller first posts a reserve price $p_t \in [0, 1]$. Buyers then decide to wait or submit a bid $b_{it}$ such that $b_{it} \geq p_t$. If no buyer bids, the game continues to period $t + 1$. Otherwise, the game ends, the buyer submitting the highest bid wins the auction, and either pays the second highest bid or $p_t$ provided that no other buyer placed a bid. Moreover, if multiple buyers submit the highest bid, then each buyer wins with equal probability. The timing of play is illustrated in figure I. Lastly, if buyer $i$ wins the item in period $t$ and must pay $p_{it} \geq p_t$, payoffs are

i. Buyer $i$: $\delta^t(\theta_i q - p_{it})$ for common discount factor $\delta \in (0, 1)$,

ii. Buyers $j \neq i$: 0

iii. Seller: $(1 - \delta^t)\theta_s + \delta^t p_{it}$.

The rest of the section proceeds as follows. First, I illustrate the revenue maximizing equilibrium under full commitment. Next, I provide a sufficient condition ensuring that the strategy profile above remains an equilibrium when one requires the seller to be sequentially rational, i.e. he has limited commitment. I lastly discuss how this result generalizes and extends to more general settings.

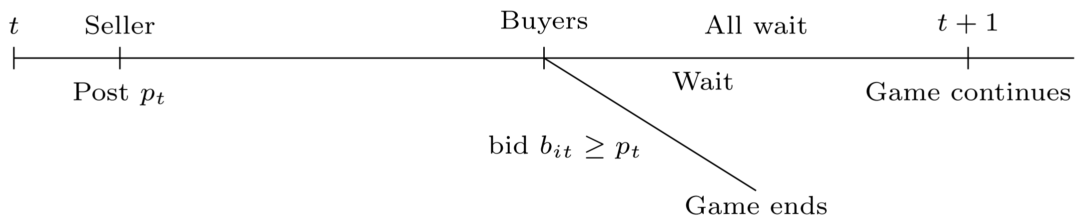



Figure I. : Timing of play at each period $t = 0, 1, \ldots$ conditional on the item remaining unsold.

### *B. Full Commitment Benchmark*

Even though I focus on the revenues that a sequentially rational seller can attain, it is useful to first characterize the optimal strategy under full commitment. An optimal strategy for the seller is a price schedule $(p_t)$ that maximizes expected revenues in period 0. Since such price schedules are not required to maximize revenues at each history, the maximum revenues attained by a seller with full commitment cannot be less than the maximum revenue with limited commitment. Hence, the full commitment benchmark is an upper bound on the revenues that a seller can attain.

In lemma 1, I establish that the seller cannot improve posting a constant price schedule i.e., for each period $t$, $p_t = p \in [0, 1]$ almost surely. This is because the seller is impatient and (as shown below) valuations fall as the item remains unsold. Consequently, delaying when buyers submit bids both delays when payments are made and lowers the rents that the seller extracts. It is further immediate that if a price schedule $(p_t)$ does not prompt buyers to delay when they submit a bid, then fixing prices at $p_0$ nets the seller the same revenues. This implies that the seller cannot improve upon running the optimal, static auction in period 0 and never re-offering his good again. Let $p^* \geq \theta_s$[1] be the reserve price associated with the optimal static auction, then an optimal strategy for the seller is $(p_t = p^*)$.

Next, when the seller fixes prices at $p^*$, buyers behave myopically. This implies that buyers expect to only submit a bid in period 0 and do so if and only if (iff) their valuation is greater than $p^*$. But what are buyer valuations? Buyers bid their valuation conditional on winning since buyers face a winner's curse. This means that buyer $i$ is more likely to win if $i$ observed

[1]It is immediate that the optimal reserve price $p^* \geq \theta_s$; otherwise, the seller sells his painting to a buyer for strictly less than he values the painting with a strictly positive probability.

good news (i.e., $x_i = 1$) and most peers $j$ observed bad news ($x_j = 0$). Since $E[q|x_i = 0] < \theta_s$, then a buyer $i$ observing good news expects to outbid each peer $j$ is $j$ observed bad news or he observes good news and a private value that is less than $\theta_i$ i.e.,

$$w_i \equiv \theta_t E[q|x_i = 1, \forall j \neq i, x_j = 0 \text{ or } x_j = 1, \theta_j \leq \theta_i].$$

I focus on symmetric equilibria for which this is the unique equilibrium. If one allows the buyers to play asymmetric strategies, revenues are lower but this is an well understood issue that is orthogonal to this paper's contribution. I now states the following lemma, which formalizes the results described above.

LEMMA 1 (First Best): *When the seller has full commitment, he posts $p_t = p^*$ in each period $t$ and each buyer $i$ bids $w_i$ in period 0 iff $w_i \geq p^*$.*

The argument behind lemma 1 is standard and I delegated its prove to appendix A. What matters is that a buyer submits a bid in period 0 iff they observe $x_i = 1$ and a private value $\theta_i$ that is above a cutoff $\theta^* \geq \theta_s$. I further find that the cutoff $\theta^*$ is strictly increasing in $\theta_s$, because buyers submitting bids (in equilibrium) must value the good more than the seller.

### *C. Limited Commitment*

I now present a condition ensuring that the strategy profile in lemma 1 is sequentially rational. To do so, I first conjecture agents follow the strategy profile in question. I then characterize how buyers update their beliefs regarding $q$ upon learning that the item failed to sell. Lastly, I derive a condition on primitives ensuring that the seller values the good more than all buyers in period $t \geq 1$ and hence he keeps his good.

Suppose that the painting failed to sell in period 0 and agents follow the strategy profile in lemma 1. Each buyer $i$ then infers that his peers waited. Buyer $i$ expects that buyer $j \neq i$ either received good news but his private value was below $\theta^*$ or he received bad news. Buyer $i$ then derives a new valuation, i.e. $v_{i1} = \theta_i E_1[q|x_i = 1]$, via Bayes rule. To do so, he first determines the initial probability that his peers waited. Since conditional on $q$ signals and private values are drawn independently, the probability that $j \neq i$ waits is

$$Pr(j\ waits|q) = w(\theta^*, q) \equiv \underbrace{Pr(x_j = 0|q)}_{\text{Bad news}} + \underbrace{Pr(x_j = 1|q)}_{\text{good news}} \underbrace{Pr(\theta_j \leq \theta^*)}_{\text{Low } \theta_j}$$

$$= \begin{cases} (1-\pi) + \pi\theta^* & q = 1 \\ \pi + (1-\pi)\theta^* & q = 0. \end{cases}$$

Since $\pi > \frac{1}{2}$ and $\theta^* < 1$, buyer $j$ is more likely to wait when $q = 0$ than when $q = 1$. Further notice that, conditional on $q$, signals and private values are drawn iid across buyers. Hence, $i$ expects that, conditional on $q$, each pair of buyers $j, k \neq i$ decided to wait independently. Thus, the probability that his peers waited was $w(\theta^*, q)^{n-1}$. By Bayes rule, $i$'s period 1 valuation is

$$(1) \quad v_{i1} \equiv \theta_i Pr(q = 1|x_i = 1, no\ trade\ at\ t = 0) = \frac{\theta_i \pi \lambda w(\theta^*, 1)^{n-1}}{\pi\lambda w(\theta^*, 1)^{n-1} + (1-\pi)(1-\lambda)w(\theta^*, 0)^{n-1}}.$$

In what follows, I provide a condition ensuring that the seller values the good more than all buyers, i.e. $v_{i1} \leq \theta_s$ for each buyer $i$. This implies that the strategy profile in lemma 1 is sequentially rational.

Sufficient Condition. — I now provide a condition for when the seller decides to keep his good after period 1. Intuitively, the seller keeps his painting provided that he does not expect to extract more rents from buyers than his valuation for the good, i.e. $\theta_s$. This is certainly true when no buyer in period 1 values the good more than the seller. In such case, it holds that:

LEMMA 2 (Learning to Commit): *A sufficient condition for the seller to optimally posts $p_t = p^*$ in each period $t$ is that*

$$(2) \qquad \frac{Pr(q = 1|x_i = 1, no\ trade\ at\ t = 0)}{Pr(q = 0|x_i = 1, no\ trade\ at\ t = 0)} \equiv \left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w(\theta^*, 1)}{w(\theta^*, 0)}\right]^{n-1} \leq \frac{\theta_s}{\theta^* - \theta_s}.$$

This condition states that the seller avoids re-offering the good provided that the event in which all buyers wait is sufficiently informative of $q$. Informally, it means that the seller avoids re-offering his good, because learning among buyers lowers valuations enough to expunge the seller's gains from trade.

### *D. Discussion*

I now discuss how and why the Coase conjecture fails in settings with interdependent values. Firstly, negative selection in the demand pool persists, i.e. high value buyers are more likely to bid than their low value peers, and prompts the seller to unprofitably screen his buyers. Secondly, negative selection further prompts buyers to screen each other, and this lowers valuations. When the second effect is sufficiently stark, the seller avoids screening his buyers and implements his optimal strategy.

In general, screening among buyers ensures that the Coase conjecture fails when the seller's valuation lies in the interior of potential buyer valuations. The seller stops re-offering his item in finite time, which this ensures that equilibrium revenues are unique and greater than immediately running an efficient auction. The intuition is threefold. Firstly, the seller still screens his buyers, i.e. if the item fails to sell, agents learn that valuations lie below a falling cutoff. Second, buyer valuations, themselves, fall over time. Lastly, the seller increasingly expects that there are fewer buyers who value the good more than himself.

The rest of the paper proceeds as follows. First, section II presents the general model. I generalize the distributions of types as well as the payoff function. Furthermore, I ensure that negative selection in the demand pool occurs in equilibrium.[2] Section III then states the results in the general model. I then discuss, in section IV, an additional effect of pessimism, i.e. it ameliorates the winner's curse. Next, I present the literature review in section V and discuss the assumptions, results, and extensions in section VI. Lastly, appendix C illustrates that the results herein extend to durable goods markets.

## II. Model

This section presents the model. First, I present the primitives, i.e. types, distributions of types, payoffs, and the seller's valuation. Next, I introduce the timing of the game. Lastly, I define strategies and equilibrium.

[2]In order to test whether learning among buyers prevents the Coase conjecture, it must be the case that there is negative selection in the demand pool; otherwise, the conjecture can fail due to non-learning factors.

### *A. Primitives*

A seller offers a single, indivisible item to $n \geq 2$ buyers. Each buyer $i$ has a type $\tau_i \in \mathcal{T} \equiv [0,1]^2$. Buyer $i$'s type (i.e., $\tau_i$) consists of a private value $\theta_i \in [0,1]$ and an interdependent value $x_i \in [0,1]$. All random variables (i.e., $(\theta_i, x_i)_i$) are drawn pairwise independently where each private value is distributed given a CDF $K$ and the interdependent values given a CDF $F$. I further assume that the CDFs $K$ and $F$ admit PDFs $k \gg 0$ and $f \gg 0$. Next, if buyer $i$ has type $\tau_i$ and other buyers have interdependent values $x_{-i} \in [0,1]^{n-1}$, the $i$'s payoff from owning the item is $u(\tau_i, x_{-i})$. I assume that the payoff function $u : [0,1]^{n+1} \to [0,1]$ satisfies the following regularity conditions stated in assumption 1.

ASSUMPTIONS 1: *The payoff function $u(\cdot)$ is strictly increasing; continuously differentiable; satisfies $u(0,0,\ldots,0) = 0$ and $u(1,1,\ldots,1) = 1$; log-supermodular; and it is symmetric i.e., for each tuple $(\tau_i, x_{-i})$ and permutation $\sigma(\cdot)$, it holds that*

$$u(\tau_i, x_{-i}) = u[\sigma(\tau_i, x_{-i})]. \tag{3}$$

The assumptions 1 are regularity conditions ensuring that the results herein are not driven by obscure, technical assumptions. There are some assumptions, however, that require some clarification. First, log-supermodularity is a standard assumption ensuring that for each player $i$, type $\tau_i$ and other players' interdependent component $x_{-i}$ enter payoffs as complements. Meanwhile, I assume symmetry of payoffs for exposition since most results would go through requiring more involved formal arguments.

Next, I ensure that there are gains from trade; otherwise, the point of the paper is moot. If the seller never expects to gain from auctioning his good, then it will vacuously holds that he never auctions the good in the first place. Now, suppose that the revenue maximizing, static auction has a reserve price of $p^* \in [0,1]$ in which buyers bid their valuation conditional on winning i.e.,

$$w_{i0} \equiv E_0[u(\tau_i, x_{-i})|\tau_i,\ i \text{ wins}]$$

where for each type $\tau_i$, the expectation is taken with respect to the initial distributions $K$

and $F$. Each buyer $i$ further expects that each buyer $j(\neq i)$ bids $w_{j0}$ iff $w_{j0} \geq p^*$; otherwise, buyer $j$ waits and does not place a bid. It is the useful to define the CDF of valuations ($w_{i0}$) as $H_0$ and its PDF as $h_0$. I can now make the assumption ensuring that there are (initially) gains from trade.

ASSUMPTIONS 2: *Let the seller's valuation for the good be $\theta_s \in (0,1)$, then I assume that the probability that some buyer is willing to bid more than the seller's valuation for the good at the optimal static auction is strictly positive but that some buyers would be willing to bid less than the seller i.e.,*

$$H_0(\theta_s) < 1.$$

### *B. Timing and Payoffs*

After establishing the setting's primitive, I now state the timing of play. I assume the most standard timing possible in order to ensure that the results herein are not driven by non-standard assumptions. It should further be notice that the timing is similar to the one provided in the example.

The timing of play is as follows. Nature first draws types ($\tau_i$) and privately informs $\tau_i$ to player $i$. The types are drawn as previously discussed. Next, at each period $t = 0, 1, \ldots$, the seller first announces a reserve price $p_t \in [0, 1]$. Each buyer $i$ then decide to wait or submit a bid $b_{it} \geq p_t$. I assume that buyer submit bids at the same time. If buyers wait (i.e., no buyer $i$ submitted some bid $b_{it}$), the game continues to period $t + 1$. Otherwise, the game ends, the buyer submitting the highest bid wins the auction, and either pays the second highest bid or $p_t$ provided that no other buyer placed a bid. Moreover, if multiple buyers submit the highest bid, then each buyer wins the auction with equal probability. The timing of play is illustrated in figure I. Lastly, if buyer $i$ wins the auction in period $t = 0, 1, \ldots$ and must pay $p_{it}$, payoffs are

i. Buyer $i$: $\delta^t[u(\tau_i, x_{-i}) - p_{it}]$ for a common discount factor $\delta \in (0, 1)$

ii. Buyer $j \neq i$: 0

iii. Seller: $\delta^t p_{it} + \sum_{s=0}^{t-1}(1-\delta)\delta^s\theta_s = \delta^t p_{it} + (1-\delta^t)\theta_s$.

Note that if the time never sells, then each buyer $i$'s payoff equals to 0 and the seller nets a payoff of $\theta_s$. Moreover, it is important to note that I assume that types are drawn only once at the beginning of the game.

### *C. Strategies and Equilibrium*

I now define histories, strategies, and equilibrium. Informally, a history is a record of all previous reserve prices since any bid would have ensured that the game ended. A strategy for the seller is then a map from histories to reserve prices and a strategy for buyers is a mapping from histories and current reserve prices to a decision to wait or which bid to submit. Meanwhile, an equilibrium imposes that beliefs are derived using Bayes rule and that players are sequentially rational.

I first define histories. In period 0, assume a set of histories $\mathbf{H}_0$ with a single null history. In period $t = 1, 2, \ldots$, however, a history $h_t$ details the preceding reserve prices i.e., $h_t = \{p_\tau\}_{\tau=0}^{t-1}$. The set of period $t$ histories is $\mathbf{H}_t \equiv [0, 1]^t$.

Next, I define strategies. A seller strategy is a collection of functions $(p_t)$ $\forall t, p_t : \mathbf{H}_t \to [0, 1]$ such that at each period $t$ and history $h_t$, $p_t(h_t)$ denotes the reserve price that the seller posts. Note that I assume that players follow pure strategies for exposition.[3] It is possible to consider behavioral and mixed strategies, but such extensions would be orthogonal to the point of this paper.

I now define a buyer strategy. A strategy for buyer $i$ consists of a collection of functions $(b_{it})$, $\forall t, b_t : \mathbf{H}_{t+1} \times \mathcal{T} \to [0, 1] \cup \{wait\}$, such that at each period $t$, history $h_t$, current reserve price $p_t$, and type $\tau_i$, it holds that $b_{it}(\tau_i, h_t, p_t)$ denotes a bid choice or a choice to wait. I assume that a buyer that is indifferent between bidding and waiting will submit a bid and in the case that a bidder is indifferent between multiple bids, he submits the highest possible bid. Next, I assume a monotonicity requirement on strategies i.e., for each period $t$, type $\tau_i$, history $h_t$, and price $p_t$ if $b_{it}(\tau_i, h_t, p_t) \neq$wait, then for each period $s \geq t$ and history $h_s$ such that $(h_t, p_t) \subset h_s$, it holds that for each price $p_s \in [0, 1]$ $b_{is}(\tau_i, h_s) \neq$wait. This condition implies that strategies are consistent with past decisions to participate in the auction. Lastly, beliefs are history dependent

[3] All functions are be assumed Lebesgue measurable. Furthermore, Liu et al (2019), Fundenberg, Levine, and Tirole (1985), and others find that almost surely neither the seller or buyers play a mixed strategy.

joint measures on $(\tau_i)_{i=1}^n$.

Now that strategies and beliefs have been defined, I can define equilibrium. The paper focuses of perfect Bayesian equilibrium (PBE) so as to not deviate from the preceding literature.

DEFINITION 1 (PBE): *A Perfect Bayesian Equilibrium (PBE) is a collection of strategies, $(p_t, (b_{it}))$, and beliefs such that for every period $t$ and history $h_t$*

*i. Given beliefs, strategies are sequentially rational*

*ii. Beliefs are derived via Bayes rule whenever possible.*

## III. Results

I now state my results. The paper's main result is that equilibrium revenues are unique and greater than immediately running an efficient auction, i.e. setting $p_0 = \theta_s$. I first present three auxiliary results. First, I prove that buyers follow a threshold bidding strategy. This means that buyers bid their valuation conditional on winning the good iff it lies above a time dependent cutoff. Next, this result implies the second result: progressive pessimism. Pessimism implies that in every period, prior beliefs likelihood ratio dominates their Bayes posteriors. This technical result drives all subsequent results in the paper.

I lastly find bounds on equilibrium revenues. First, I prove that an upper bound on equilibrium revenues. In every period, the seller cannot improve upon immediately running the revenue maximizing, static auction given his current beliefs and keeping the good if it fails to sell. Next, I find a revenue floor that is higher than immediately running an efficient auction and does not account $\delta$ or the frequency of future re-offerings. In each period, the seller's revenues are greater than running any auction where upon learning that the item failed to sell, buyers lower their valuation enough to justify that the seller keeps his good.

### *A. Auxiliary Results*

SKIMMING PROPERTY. — The first auxiliary result states that buyers play a threshold strategy in every equilibrium i.e., in every equilibrium, each player $i$ participates in an auction iff $i'$s valuation is above a history-dependent cutoff. This result imposes a tractable structure behind equilibrium beliefs. First define expectations given period $t$ beliefs as $E_t[\cdot]$ and each player $i$'s valuations given his type $\tau_i$ and the public history until period $t$ as

$$w_{it} \equiv E_t[u(\tau_i, x_{-i})|\tau_i,\ i \text{ wins}].$$

This implies that player $i$'s time $t$ valuation is the expected he expects to net conditional on winning the auction. Note that when it is useful to specify the valuation as a function of the history, I will write it as $w_{it}(h_t) \equiv E[u(\tau_i, x_{-i})|h_t, \tau_i,\ i \text{ wins}]$. Next, I can define a threshold strategy.

DEFINITION 2 (Threshold Strategy): *Buyer $i$ is said to play a threshold strategy iff there exists a collection of functions $(u_{it}), \forall t\ u_{it} : \mathbf{H}_{t+1} \to \Re$ such that $i$ bids his valuation in period $t$ and history $h_t$ if $w_{it}(h_t) \geq u_{it}(h_t)$; otherwise, $i$ waits.*

This definition states that buyer $i$ follows a threshold strategy if he bids his valuation conditional on winning provided that it lies above a cutoff. Otherwise, buyer $i$ waits. Note that buyers bid their valuation conditional on winning is a standard argument presented Myerson (1981) and Krishna (2004). The following lemma characterizes buyers' equilibrium behavior.

LEMMA 3 (Skimming Property): *In every PBE, each buyer $i$ plays a threshold strategy.*

The proofs are in the appendix, but I sketch the argument below. For each buyer $i$, his type $(\tau_i)$ is unverifiable. This implies that when $i$ observes a type $\tau_i$, he can implement the strategy associated with observing type $\tau_i'$ without the possibility of being detected. This implies that the difference in payoffs between participating in the auction relative to the payoffs he nets from waiting must be increasing in buyer $i$'s current valuation. This establishes the skimming property.

This result is key for two reasons. First, this result implies that there still exists negative selection in the demand pool. This negative selection ensures that beliefs evolve as described in the next auxiliary result. Secondly, if buyers' decision to participate at auction follows a cut-off rule as described above, then the following auxiliary results proceed regardless of the auction protocols considered. How buyers bid and interact with each other is inconsequential when it comes to characterizing learning from the item failing to sell. The auction format, nonetheless, matters since it determines who participates in the optimal, static auction available to the seller. I delegate this discussion to the appendix to avoid the issue of presenting a general auction format in the main text.

Principle of Progressive Pessimism. — The next result characterizes learning. Heuristically, the seller and buyers become increasingly pessimistic regarding other buyers' (IV) components and valuations. This is the key insight to my main result. I first present a standard ordering on distributions and state my result using such ordering. A CDF $H[0,1]$ likelihood ratio dominates another CDF on $G[0,1]$ provided that the CDF $G$ systematically gives higher weight to lower realization of a random variable than $H$. The formal definition is the following.

DEFINITION 3 (Likelihood Ratio Dominance): *Suppose that $H[a,b], a < b$, and $G[a,b]$ are CDFs admitting pdfs $h \gg 0$ and $g \gg 0$, respectively. Then, $H$ likelihood ratio dominates $G$, i.e. $H \trianglerighteq G$, if for each pair of $x, x' \in [a,b]$ such that $x \leq x'$, it holds that*

$$\frac{g(x')}{g(x)} \leq \frac{h(x')}{h(x)}. \tag{4}$$

This is a strong notion of stochastic dominance. It implies first- and second-order, hazard and inverse hazard rate, stochastic dominance. Indeed, if one has a sequence of random variables $(x_n)$ and for each $n$, it holds that the CDF of $x_n$ likelihood ratio dominate the CDF of $x_{n+1}$, then the sequence is stochastically decreasing, i.e. it is increasingly likely that one observes high realizations as $n$ increase and less likely to observe high realizations.

Next, define for each period $t$ the CDFs $F_t, K_t$, and $H_t$ as the equilibrium path beliefs regarding each buyer's interdependent value, private value, and valuation as of the beginning of the period. Note that $F_t(.)$, for example, refers to the seller or a buyer $i$'s beliefs regarding buyer $j \neq i$'s value $x_j$. I now present the result.

THEOREM 1 (Progressive Pessimism): *In every PBE, the expected valuations and types are stochastically decreasing:*

$$\forall t, \quad F_t \trianglerighteq F_{t+1}, G_t \trianglerighteq G_{t+1}. \tag{5}$$

I now state an immediate corollary, i.e. valuations and the expected dispersion of valuations falls.

COROLLARY 4 (Expected Dispersion in Valuations Falls.): *In every PBE and period $t$, the dispersion in valuation falls, i.e. for every pair of types $\tau, \tau'$, it holds that*

$$E_t[|v_t(\tau) - v_t(\tau')|] \geq E_{t+1}[|v_t(\tau) - v_t(\tau')|]. \tag{6}$$

*Next, for every period $t$, buyer $i$, and type $\tau_i$, it holds that $v_t(\tau_i) \geq v_{t+1}(\tau_i)$.*

These results allow me to decompose learning in the current IV setting and compare it to a setting with private values. Such comparison clarifies the role. For exposition, I focus on beliefs regarding valuation, i.e. $v_{it} = v_t(\tau_i)$. First, let $v_t$ be the smallest valuation for which $H_t(v_t) = 1$, i.e.

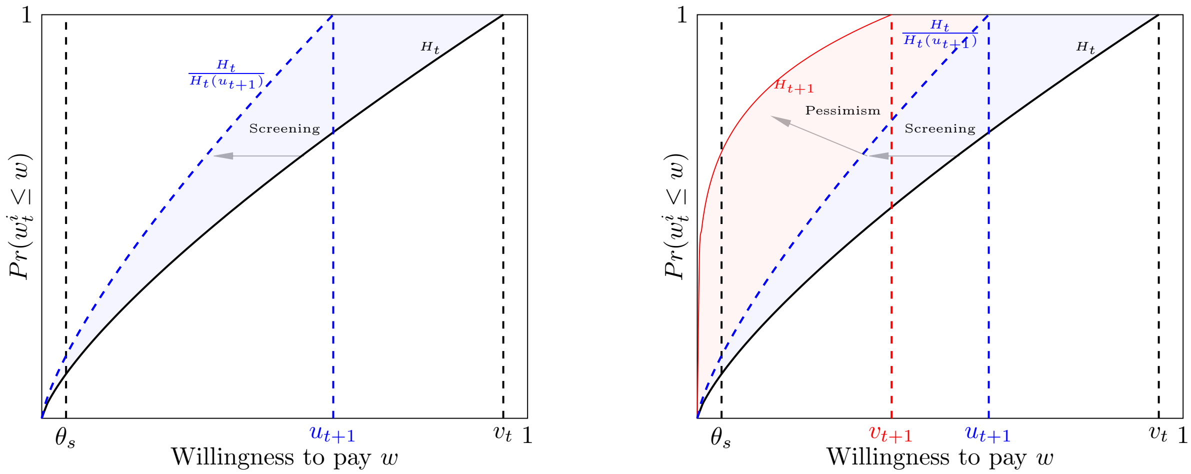
$$v_t \equiv \inf\{x \in [0,1] | H_t(x) = 1\}$$

Note that $v_t$ is non-increasing in $t$ due to the skimming property. In every period $t$, agents expect that buyers have valuations below $v_t$ and are distributed by the CDF $H_t[0, v_t]$.

If the item fails to sell, agents learn that buyers have a valuations below a cutoff $u_{t+1} \leq v_t$. Note that in a comparable private value setting, the seller's beliefs regarding buyer valuations in period $t+1$ is given by the CDF $\frac{H_t}{H_t(u_{t+1})}[0, u_{t+1}]$. Next, every buyer further lowers their valuation in response to their peers lack of trade and hence the maximum valuation in period $t+1$ is $v_{t+1} \leq u_{t+1}$. Figure II illustrates the decomposition described above.

The proof of theorem 1's is inductive. Intuitively, when a buyer $i$ observes that his peer $j \neq i$ waited, he learns that $v_{jt} \leq u_t$ and expects that for each pair of values $x_j$ and $x'_j$ where $x_j \leq x'_j$, there exists more potential values $\theta_j$ for which the expected valuation of $(x_j, \theta_j)$ lies below $v_t$ than for realizations. Consequently, the probabilities ratio is non-increasing is $x_j$. The stochastic dominance discussed above follows immediately from this observation and argument extends to beliefs regarding private values as well as for valuations.

Bound on Revenues. — I now derive bounds on equilibrium revenues. First, I define an upper bound on revenues. In each period, I find that the seller cannot improve upon immediately




(a) *Intra-period Belief update.* (b) *Inter-temporal Belief Updating.*

Figure II. : Equilibrium belief decomposition in every period $t$: Skimming, Progressive Pessimism, valuation re-assessment.

running the revenue maximizing auction given the information at hand and never re-offering his good. Remember that re-offering his good is often sequentially rational. Next, I derive a lower bound. Intuitively, I calculate the revenue maximizing auction after which the seller can commit to keep his good.

First, suppose that after period $t$, the seller commits to his optimal continuation strategy. What strategy would he pick? I show that the seller cannot improve upon immediately running the static optimal auction given the information he holds in period $t$ and keeping the good if it fails to sell. If $p_t^*$ denotes the optimal, static auction's reserve price, then the seller prevents re-offering his good by setting $p_s = p_t^*$ for every period $s \geq t$ with probability 1. Next, I derive an expression for the revenues attained by a static auction. A static auction held in period $t$, when beliefs are $H_t[0, v_t]$, and a reserve price of $p \in [0, 1]$ nets the seller an expected revenue of

$$r(p, H_t) \equiv \theta_s + E_t[\chi(v_t^2 \geq p)\{\bar{\phi}(v_t^2, H_t) - \theta_s\}]$$

where $v_t^2$ is the second highest valuation among buyers and $\bar{\phi}(x, H_t)$ are the ironed out virtual values as given by the current distribution of valuations conditional on winning the auction. Furthermore, the optimal static auction's revenues are denoted as $r_t^*$ and satisfy

$$r_t^* \equiv \max_{p \in [0,1]} r(p, H_t). \tag{7}$$

Next, I find a period 0 revenue floor. The seller may not be able to implement his revenue maximizing auction and never re-auction his good, but there exist some auctions that the seller could run in period 0 whereupon learning that the item failed to sell, buyers end up willing to bid less than the seller's valuation. First define valuations conditional on the seller running a static auction in period 0 with a reserve price of $p$, buyers bid myopically, i.e. each buyer $i$ bids iff $w_0(\tau_i) \equiv E[u(\tau_i, x_{-i})|i\ wins] \geq p$, and yet the good remains unsold as

$$w_0(\tau_i, p) \equiv E[u(\tau_i, x_{-i})|i\ wins,\ \forall j \neq i, w_0(\tau_j) \leq p].$$

I present the lower bound below.

DEFINITION 4 (Commitment Auctions): *In every PBE, the price $p \in [0, 1]$ defines a commitment auction if for every type $\tau_i$ such that $w_0(\tau_i) \leq p$, it holds that $w_0(\tau_i, p) \leq \theta_s$. Next, $\underline{r}_0$ is a revenue floor that satisfies*

$$\underline{r}_0 = \max_{p \in [0,1]} r(p, H_t)\ s.t.\ \forall \tau \in \mathcal{T},\ s.t.\ \ w_0(\tau) \leq p, w_0(\tau, p) \leq \theta_s. \tag{8}$$

It should be noted that the set of such auctions is non-empty as $p = \theta_s$ is a commitment auction. Next, denote the expected revenues from running an efficient auction in period 0 as $r_0^e$ and the equilibrium revenues in period $t$ as $r_t$. Also, a CDF $H[a, b]$, for $a < b$, is regular iff for each $x$, the function $x - [1 - H(x)]/h(x)$ is increasing. I now state the subsequent theorem.

THEOREM 2 (Coase fails): *In every PBE, revenues are below the optimal, static auction revenues, i.e. $\forall t,\ r_t \leq r_t^*$. Meanwhile, if $H_0(\cdot)[0, 1]$ is a regular distribution, then the seller's equilibrium revenues are strictly higher than immediately running an efficient auction, namely $r_0^e < \underline{r}_0 \leq r_0$.*

I first make some comments before sketching the proof. The revenue floor is independent of how often the seller offer his good and the discount factor. Therefore, the Coase conjecture does not hold in this setting. Lastly, this lower bound need not bind when types are two-dimensional. However, in the accompanying paper, Ramos-Mercado (2022), I prove that when the types are one-dimensional, this revenue floor is binding.

The proof proceeds in three steps. First, suppose that after some history, the seller offers and commits to a dynamic, trade mechanism that only depends on each buyer's current valuation. This mechanism consists of an allocation and payment rules as well as a time when trade and payoffs are realized such that the outcome can be implemented with second-price auctions with reserve prices. I show that the seller might as well focus on mechanisms in which he only trade immediately and if the item fails to sell, the seller keeps the item from henceforth.

The argument follows a replication argument. Fix some individually rational and incentive compatible, dynamic mechanism. I construct an alternative mechanism where the period in which he sells the good is either immediate or never. The proof further shows that such mechanism yields the seller the same revenues as the initial mechanism. Consequently, it is without loss of generality to focus on this restricted class of mechanisms rather than a larger class.

### *B. Main Result*

The previous results leave three questions unanswered. First, under what conditions are equilibrium revenues with limited commitment equal to revenues with full commitment? Second, are there multiple equilibria? Otherwise, it is possible for learning to mitigate revenue losses in some equilibria and not in others. Lastly, when the seller cannot implement his revenue maximizing auction, what happens?

The answers to these questions are as follows. First, the equilibrium is essentially unique, so multiplicity of equilibria does not undermine revenue predictions. Next, I find sufficient conditions ensuring that the seller implements his optimal strategy. Otherwise, I find that the game effectively ends in finite time. This means that the seller stops eliciting bids almost surely by a finite time horizon and its formal definition is below.

DEFINITION 5: *The game essentially ends in finite time iff there exists a deterministic period* $T < \infty$ *such that buyers almost surely wait in every period* $t \geq T$ *and PBE.*

I can now state the paper's main theorem.

THEOREM 3: *The PBE is essentially unique and the game essentially ends in finite time. Furthermore, revenues equal the optimal, static auction revenues, i.e.* $r_0 = r_0^*$*,if the following condition holds:for every type* $\tau_i$ *such that* $w_0(\tau_i) \leq p^*$*, it holds that*

$$w_0(\tau_i) - v(\tau_i, p^*) \geq \frac{1 - H_0[w_0(\tau_i)]}{h_0[w_0(\tau_i)]}. \tag{9}$$

I now discuss theorem 9. First, the seller commits to the static, optimal auction iff all excluded buyers from the optimal auction demand smaller information rents than the loss of valuation due to learning. Otherwise, revenues with limited commitment are lower than revenues with full commitment. However, the seller stops eliciting bids in finite time.

The proof has two steps. First, there exists a deterministic period $T < \infty$ such that in all PBE, the seller can fix prices after period $t$. Intuitively, the seller is sequentially rational, impatient, and can always implement his revenue maximizing commitment auction. But as the item fails to sell, the maximum valuation falls enough by period $T$ such that it is optimal to implement an auction that prevents re-offerings. The second part of the proof characterizes precisely when the seller implements his optimal strategy. I use the virtual value characterization of the optimal reserve price and the observation that no buyer, in period 1, should be willing to bid more than the seller's valuation.

## IV. Bid Shading Reduction

Progressive pessimism implies that as buyers learn from each other, valuations fall and the seller becomes increasingly pessimistic about finding a buyer who values the good more than himself. I also find that pessimism ensures that the winner's curse is ameliorated over time. The winner's curse is the observation that buyers do not bid their valuation but their valuation conditional on winning the good. In this section, I show that as the game progresses the ratio between the two increases.

I return to the model from section I and, for exposition, assume that the seller can only offer his good twice. This allows me to derive the optimal reserve prices via backwards induction in appendix B. I then derive valuations, bids (valuations conditional on winning), and the ratio between the two. This mechanical calculation illustrates the result.

Intuitively, each buyer expects that they are more likely to win when the common value is low rather than high. But as the game progresses, each buyer expects that it is increasingly likely that they outbid their peers, regardless of the $q$. Winning the auction hence becomes increasingly uninformative of $q$ and bids stop reflecting such considerations.

### *A. Social Learning in Sequential Second-Price Auctions*

Deriving Valuations. — I now derive the valuations of buyers observing good news. Notice that buyers observing bad news will not bid, so I only discuss their valuations in appendix B. Suppose that buyer $i$ observed good news and a private value of $\theta_i$ in period 0, then his valuation is $v_{i0} = \theta_i E_0[q|x_i = 1]$. Since $q \in \{0, 1\}$, it holds that $E_0[q|x_i = 1] = Pr_0(q = 1|x_i = 1)$ and Bayes rule implies that his valuation is

$$v_{i0} = \theta_i \left[\frac{\lambda\pi}{\lambda\pi + (1-\lambda)(1-\pi)}\right] = \theta_i \left[\frac{\hat{\lambda}\hat{\pi}}{\hat{\lambda}\hat{\pi} + 1}\right]$$

for $\hat{\gamma} \equiv \gamma/(1-\gamma)$ and $\gamma = \pi, \lambda$. The second equality derives $Pr(q = 1|x_i = 1)$ as a strictly increasing function of the likelihood ratio, i.e. $r_0(x_i = 1) \equiv Pr(q = 1|x_i = 1)/Pr(q = 0|x_i = 1) = \hat{\pi}\hat{\lambda}$ and

$$Pr(q = 1|x_i = 1) = \frac{r_0(x_i = 1)}{r_0(x_i = 1) + 1}.$$

Next, let us derive buyer $i$'s valuation in period 1. If the item failed to sell in period 0, buyer $i$ learns that other buyers $j \neq i$ either received bad news (i.e. $x_i = 0$) or observed good news ($x_j = 1$) and a private value below a cutoff of $\theta_1 \in [\theta_s, 1]$. As in section I, buyer $i$ believes that his peers waited, conditional on $q$, with probability $w(\theta_1, q)^{n-1}$; where $w(\theta_1, q)$ was derived in equation I.C. This implies that the likelihood ratio in period 1 equals to

$$r_1(x_i = 1) \equiv \frac{Pr(q = 1|x_i = 1, no\ bids)}{Pr(q = 0|x_i = 1, no\ bids)} = \hat{\pi}\hat{\lambda}\left[\frac{w(\theta_1, 1)}{w(\theta_1, 0)}\right]^{n-1} \leq r_0(x_i = 1).$$

The inequality above follows from the fact that the good is more likely to not sell when $q$ is low rather than low. Consequently, buyer $i$ updates his valuation to $v_{i1} = \theta_i E_1[q|x_i = 1]$ via Bayes rule and it equals to

$$v_{i1} = \theta_i \left[\frac{r_1(x_i = 1)}{r_1(x_i = 1) + 1}\right] \leq v_{i0}.$$

Deriving Bids. — I now derive each buyer's potential bid in each period $t = 0, 1$. If buyer $i$ submits a bid in period 0, he expects to win a higher probability when $q = 0$ rather than

$q = 1$. In equilibrium, buyer $i$ internalizes this observation and bids his valuation conditional on winning the good, i.e. he bids

$$b_{i0} = w_{i0} \equiv \theta_i E_0[q|x_i = 1,\ i\ wins].$$

As before, $E_0[q|x_i = 1,\ i\ wins] = Pr_0[q = 1|x_i = 1,\ i\ wins]$, but it is not immediate what this belief happens to be. Deriving the likelihood ratio, however, makes this clear.

I now derive the probability that $i$ wins the auction given $q$. Buyer $i$ wins iff he outbids each peer $j \neq i$. In appendix B, I show that buyers observing bad news wait; meanwhile, buyers observing good news follow a bidding strategy that is strictly increasing in the private value. This means that if buyer $i$, observing good news, outbids his peer $j \neq i$ it is either because $j$ observed bad news or a private value $\theta_j$ $(\leq \theta_i)$. Furthermore, if the maximum private value held by buyers who observe good news is $\bar{\theta}$, then the probability that $i$ wins the auction, given $q$, is

$$W(\bar{\theta}, \theta_i, q) \equiv Pr(x_i = 0|q) + Pr(x_i = 1|q)Pr(\theta_j \leq \theta_i|\theta_j \leq \bar{\theta}) = \begin{cases} (\frac{\theta_i}{\bar{\theta}})\pi + (1-\pi) & if\ q = 1 \\ (\frac{\theta_i}{\bar{\theta}})(1-\pi) + \pi & if\ q = 0. \end{cases}$$

Since $\pi \in (1/2, 1)$,$i$ is more likely to win when $q$ is low rather than high. Also, in period 0, it was assumed that $\bar{\theta} = 1$, so buyer $i$'s expected odds of winning are $W(1, \theta_i, q)$ for a given $q$. Hence, the period 0 likelihood ratio conditional on winning, i.e. $r_0(x_i = 1, i\ wins)$, is

$$r_0(x_i = 1, i\ wins) = r_0(x_i = 1)\Bigg[\underbrace{\frac{W(1, \theta_i, 1)}{W(1, \theta_i, 0)}}_{(<1)\text{ Winner's Curse.}}\Bigg]^{n-1}$$

This implies that buyer $i$ is willing to bid

$$b_{i0} = \theta_i\left[\frac{r_0(x_i = 1, i\ wins)}{r_0(x_i = 1, i\ wins) + 1}\right].$$

Next, let us calculate buyer $i$'s bid in period 1. Buyer $i$ still bids his valuation conditional on winning the good, but he now knows that if buyer $j \neq i$ observed good news, then $\theta_j \leq \theta_1$ with probability 1. This means that $\bar{\theta} = \theta_1 \leq 1$ and the odds that buyer $i$ outbids each buyer $j$ is $W(\theta_1, \theta_i, q)$ given $q$. Since private values are $q$-conditionally independent, buyer $i$ arrives at the following likelihood ratio

$$r_1(x_i = 1, i\ wins) = r_1(x_i = 1)\left[\frac{W(\theta_1, \theta_i, 1)}{W(\theta_1, \theta_i, 0)}\right]^{n-1} \leq r_1(x_i = 1).$$

Consequently, buyer $i$ bids $b_{i1} = w_{i1} \equiv \theta E_1[q|x_i = 1, i \text{ wins}]$ which equals to

$$b_{i1} = \theta_i\left[\frac{r_1(x_i = 1, i\ wins)}{r_1(x_i = 1, i\ wins) + 1}\right].$$

The following result follows from the observation that the ratio of likelihood ratios (i.e. $r_t(x_i = 1, i \text{ wins})/r_t(x_i = 1)$) increases over time:

$$\frac{r_1(x_i = 1, i\ wins)}{r_1(x_i = 1)} \geq \frac{r_0(x_i = 1, i\ wins)}{r_0(x_i = 1)}.$$

*B. Bid Shading falls.*

I now prove that bid shading falls over time, i.e. $b_{it}/v_{it}$ is increasing in $t$. First, if buyer $i$ has a private value of $\theta_i$, then for each period $t = 0, 1$ one can derive the ratio $b_{it}/v_{it}$ as

$$\frac{b_{it}}{v_{it}} = \frac{r_t(x_i = 1,\ i\ wins)}{r_t(x_i = 1)}\left[\frac{1 + r_t(x_i = 1)}{1 + r_t(x_i = 1,\ i\ wins)}\right] = \frac{1 + r_t(x_i = 1)}{\left[\frac{W(\theta_t, \theta_i, 0)}{W(\theta_t, \theta_i, 1)}\right]^{n-1} + r_t(x_i = 1)}. \tag{10}$$

The second equality follows from unraveling the likelihood ratio conditional on winning the good. Next, in period $t = 1$, the likelihood ratio equals the likelihood ratio in period $t - 1$ times the relative odds that the item fails to sell in period $t - 1$ as a function of $q$. Meanwhile, the period 1 likelihood ratio conditional on winning can be unraveled in a similar fashion by embedding the fact that the good failed to sell in period 0. If one plugs these observations into equation 10, it holds that

$$\frac{b_{it}}{v_{it}} = \frac{1 + r_t(x_i = 1)}{\left[\frac{W(\theta_1, \theta_i, 0)}{W(\theta_1, \theta_i, 1)}\right]^{n-1} + r_t(x_i = 1)} = \frac{\left[\frac{w(\theta_1, 0)}{w(\theta_1, 1)}\right]^{n-1} + r_{t-1}(x_i = 1)}{\left[\frac{W(\theta_1, \theta_i, 0) w(\theta_t, 0)}{W(\theta_t, \theta_i, 1) w(\theta_1, 1)}\right]^{n-1} + r_{t-1}(x_i = 1)}. \tag{11}$$

Observe that the item is less likely to not sell when $q = 0$ than when $q = 1$ in every period $t$, this means that $w(\theta_t, 0) \geq w(\theta_t, 1)$ for each period $t$. Similarly, buyer $i$ is more likely to win when $q = 0$ than when $q = 1$ in every period $t$, so $W(\theta_t, \theta_i, 1) \leq W(\theta_t, \theta_i, 0)$. This implies that the following function is non-increasing:

$$\forall x \geq 0, f(x) = \frac{x + r_{t-1}(x_i = 1)}{x\left[\frac{W(\theta_t, \theta_i, 0)}{W(\theta_t, \theta_i, 1)}\right]^{n-1} + r_{t-1}(x_i = 1)}.$$

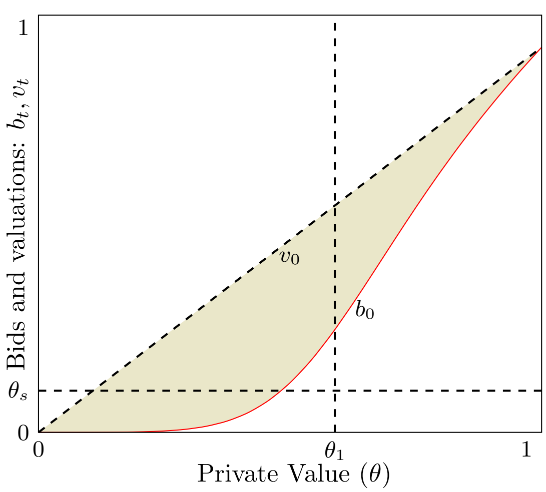
is non-increasing. Thus, it holds that for $t = 1$

$$\frac{b_{i1}}{v_{i1}} \geq \frac{1 + r_0(x_i = 1)}{\left[\frac{W(\theta_1, \theta_i, 0)}{W(\theta_1, \theta_i, 1)}\right]^{n-1} + r_0(x_i = 1)} \geq \frac{1 + r_{t-1}(x_i = 1)}{\left[\frac{W(1, \theta_i, 0)}{1, \theta_i, 1)}\right]^{n-1} + r_{t-1}(x_i = 1)} = \frac{b_{i0}}{v_{i0}} \tag{12}$$

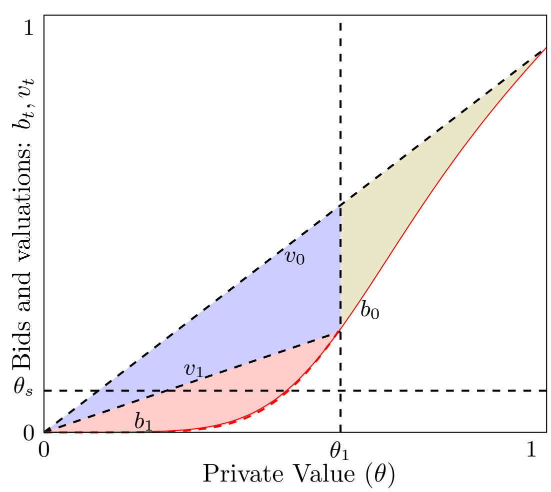
This last inequality states the main result of this example, i.e. bids relative to valuations increase over time. The following lemma summarizes this result.

LEMMA 5: *If buyer $i$ observes good news ( $x_i = 1$) and a private value $\theta_i \leq \theta_1$, then $\frac{b_{i1}}{v_{i1}} \geq \frac{b_{i0}}{v_{i0}}$.*

Figure IIIb decomposes how bid shading falls over time. First, valuations among buyers fall in period 1. Potential bids also fall, but the fall by less than the valuations. The left-hand panel depicts in gold the trade surplus that the seller cannot extract from buyers in period 0. Meanwhile, the right-hand panel decomposes this region in the parts. The remaining gold region depicts the surplus loss since no buyer both observed good news and a private value which would have compelled him to bid in period 0. The purple region then describes surplus loss due to learning; whereas the red region illustrates the trade surplus that the seller cannot extract from buyers in period 1.




(a) *Bids as a function of private values in $t = 0$.*




(b) *Bids as a function the private value in $t = 0, 1$.*

Figure III. : Equilibrium bidding among buyers observing good news, i.e. $x_i = 1$, as a function of the private value.

## V. Related Literature

The Coase Conjecture (1972) illustrates the implications of limited commitment in a stylized manner. Intuitively, patient, high valuation buyers are more likely to buy the good than their low valuation peers. Thus, a sequentially rational monopolist sequentially lowers prices to trade with his remaining buyers. But this pricing behavior persuades some high valuation buyers to delay their purchase decision: thus, lowering profits. Coase conjectured that as the seller interacts with his buyers ever more frequently, then his price converges his marginal cost.

Indeed, Bond and Samuelson (1984), Gul et al (1986), Kahn (1986), and others corroborate the Coase conjecture in stationary equilibria. Subsequent papers, however, illustrate that the conjecture is not robust when either the seller follows a non-stationary strategy or when one slightly perturbs the underlying setting. Ausubel and Denekere (1989) prove that there exist non-stationary equilibria where the seller sustains a gradually declining price schedules and attains profits that are arbitrarily close to the static monopoly rents attainable under full commitment. Intuitively, agents posit a candidate price schedule and if the seller ever posts a price different from the one dictated by the schedule in question, then buyers expect that the subsequent subgames follow a Markov equilibrium. These Markov equilibria net the seller no profits in the continuous time limit. Thus, the seller prefers sticking to the candidate price schedule if he interacts with buyers sufficiently frequently.

Board and Pycia (2014), meanwhile, added outside options for buyers and found that all equilibria are payoff equivalent to fixing prices equal to the one-shot monopoly price. Heuristically, equilibrium prices are greater than the lowest value among remaining buyers: thus, remaining buyers with relatively low valuations are better off exiting the market than waiting and expecting no consumer surplus in the future. This leads buyers to either immediately purchase the good or exit the market. Thus, the seller *can* fix prices since he has no remaining buyer with whom to trade.

Fundenberg et al (1987) also find that the Coase conjecture fails when seller have an outside option, e.g. consume the good himself or sell it to another buyer. The seller only offers his good until a finite, terminal period since he eventually expects that the rents that he can extract from buyers may not compensate for him forgoing his outside option. This result differs markedly from my result because the seller offers his good to multiple buyers at the same time, via an auction. I too find that the seller eventually becomes sufficiently pessimistic and decides to not re-offer his good but learning among buyers further distorts this intuition twofold. First, learning among buyers further lowers the total surplus above and beyond what the seller learned from his peers. Secondly, social learning allows the seller to extract an increasing share of the surplus among the buyers in question. Further work by McAfee and Wiseman (2008), Madarász (2021), Bagnoli et al (1989), von der Fehr and Kuhn (1995), Montez (2013), Feinberg and Skrypacz (2005), Karp (1993), Ortner (2017) and others similarly find small that environment preventing the Coase conjecture.

Next, the Coase conjecture generalizes to auctions. Vincent and McAfee (1997) first showed that when the seller values the good strictly less than all buyers, there exists an essentially unique equilibrium, where the seller runs a sequence of standard auctions with declining reserve prices until a finite, terminal period in which he ensures that the item is sold. Their paper, however, assumes a "gap" case in a private value setting. This means that the seller who values the good the least values the good more than the seller.

Liu et al (2019) then studied the no gap case and showed that when there are at least 3 buyers and for most independent private value environments with 2 buyers, the seller might as well immediately run an efficient auction in the limit when the time between transactions go to zero. They show that with multiple buyers, one cannot construct a reputational equilibrium—as in Ausubel and Deneckere (1989)—where the seller can profitably screen his buyers and find that the unique level of profit are those attainable by the auction mentioned above.

The intuition behind the Coase conjecture further extends to contracting settings—see Skreta (2006, 2015), Doval and Skreta (2021), and others. When a principle commits only to short term contracts and interacts repeatedly with agents holding private information, he extracts information rents from his agents and changes the contracts offered. This, in general, limits the principal's ability to provide the agents with incentives and restricts implementable outcomes. Doval and Skreta (2021) for instance show that these contracts can be characterized via a generalized version of the Coase Conjecture. Meanwhile, Burzustowski et al (2021) finds that allowing the principal to implement dynamic contracts that he can void at will, however, allows him to avoid the Coase Conjecture.

## VI. Discussion and conclusion

### *A. Discussion*

In this subsection, I discuss how changes to the primitives affect the paper's results. I also discuss how results extend to more general auction and non-auction environments.

Assumptions on Primitives: Buyers. — My main results disregarded several issues pertaining preferences and the signal structure. I firstly assume that all random variables are drawn independently from each other, but types can be, for instance, affiliated across buyers. In the motivating example, see section I, this assumption allowed me to describe how each buyer learns deduces information from each one of his peers separately. In general, each buyer learns from the lack of bidding decisions of their peers, as a collective. This means that he updates his beliefs cognizant of the way the signals are jointly drawn. Aside from this distinction, learning among buyers would proceed as before.

The model further assumes symmetric buyers and this does not represent many interesting settings. For example, an art collector's valuation is more responsive to a painting's resale value than the director of a museum since the museum profits from exhibiting the piece rather than reselling it. In equilibrium, however, there still exist negative selection in the demand pool and hence the learning dynamics described herein persist.

Assumptions on Primitives: Seller. — A less innocuous assumption is that the seller's valuation is constant and in the interior of potential buyer valuations. This assumption avoids three issues. First, if all buyers value the good more than the seller for certain, i.e. $\theta_s = 0$, then the seller never stops re-offering his good in finite time. Indeed, this may seem like an important case, but I claim that it is pathological. First, for every value $\theta_s < 0$, this is a "gap" case and an argument like the one presented in Fundenberg, Levine, and Tirole (1985) shows that the seller stops re-auctioning his good in finite time. Meanwhile, I already established this precise issue when $\theta_s > 0$.

Next, the seller's valuation may depend on a buyers' interdependent components. For example, the seller may care about his artwork's resale value at a different auction. I find that the results herein still hold when the seller's valuation for the good are not as responsive to the resale value as buyers. This too can be understood with the art auction example. The seller may resale his good at the same art markets as his buyers, but he can also run a private sale. Therefore, his valuation for the good is not as responsive to a low resale value at an alternative public offering.

Lastly, the seller could also observe an interdependent value component, making him an informed principal. For example, the seller may be privately informed of the artwork's resale value and his reserve price may signal his private information. I disregard this possibility, because it adds a significant layer of complexity that obfuscates learning among buyers. Such considerations are important, and I plan to study this precise question in my future work.

Assumptions on the auction procedure.. — The paper further assumes that the seller runs second-price auctions. I make this assumption to simplify exposition and to compare results with the literature, but if the decision to participate in an auction result in negative selection among buyers, the results follow. The auction format only matters in as far as determining who would be excluded from the optimal static auction. Appendix A.A3 presents a broad collection of auction in which there exist negative selection in the demand pool and hence my result can be extended to those settings.

Is there something special about auctions?. — The last point of clarification is that the results herein persist beyond auction settings. In appendix C, I consider a durable good market with a single seller and a continuum of buyers with a common value. I prove that learning among buyers contravenes the Coase conjecture in stationary settings. This is because buyers observe the mass of consumers who previously purchased the good and this eliminates all dispersion in valuations. Thus, the seller can extract all rents from the remaining buyers.

### *B. Conclusion*

The Coase conjecture predicts that limited commitment lowers profits since the seller serially auctions his good. I, however, find that serial re-offerings of items that fail to sell is empirically rare, so Coasian dynamics unlikely to be salient. In this paper, I proved that this behavior can be rationalized in settings with interdependent IV values. Moreover, it is possible for a sequentially rational seller to attain his maximum expected revenues with full commitment.

Intuitively, learning among buyers serves as an endogenous commitment device. When the seller runs an auction and the good does not trade, the seller and buyers first learn that buyers had lower valuations than expected. Each buyer further expects that their peers' low valuations were informed, at least partially, by interdependent value, private information. Consequently, buyers lower their valuation for the good and this limits, or outright prevents, items from being serially re-offered. When valuations fall by more than buyer's initial information rents, the seller prevents his optimal strategy with full commitment.

I also characterize what happens when the seller cannot commit to his revenue maximizing auction. The seller stops offering his good in finite time, he extracts an increasing share of the trade surplus over time, and his equilibrium revenues are greater than immediately running an efficient auction. I further show that bid shading falls over time. Intuitively, buyers expect it to be increasingly likely that they outbid their peers regardless of their peer's private information. Winning the item at auction, therefore, becomes an increasingly uninformative event and buyers respond by bidding an increasing share of their valuation.

My results, lastly, illustrate that the received Coasian logic is incomplete. The Coase conjecture implies that a sequentially rational seller screens his buyers, which results in delayed market participation and lower profits. In IV settings, however, buyers also screen each other and this that as the seller learns from his buyers, he offers buyers terms of trade that increasingly favor remaining consumers. When types are interdependent, however, *buyers* also lower their willingness to pay, the dispersion in valuations falls, and the seller can extract an increasing share of the remaining trade surplus. Consequently, learning among buyers limits re-offering and the payoff relevance of sequential rationality.

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## One-shot Benchmark.

I return to the setting in section I and characterize the revenue maximizing, static auction. First, I derive the optimal, symmetric bidding strategy. Next, I state the seller's revenue as a function of his chosen reserve price and derive the optimal reserve price $p^*$.

### *A1. Symmetric Bidding Strategy.*

In this subsection, I prove that buyer participating in the auction bid their valuation conditional on winning the good in the unique, symmetric equilibrium. Note that the seller picks price $p \geq \theta_s$; otherwise, he accepts payments for his good that are below his valuation with a strictly positive probability. This implies that only buyers observing good news bid.

I first derive the valuation of a winning buyer. Suppose that the seller posted a price of $p$, buyer $i$ bid $b$, and $i$ wins. Then, if buyer $i$ expects that only his peers $j \neq i$ observing good news submit bids $b(\theta_j)$, for some strictly increasing function $b$, then the initial probability that $i$ wins conditional on $q$ is

$$w(b^{-1}(b), \theta^*, q) = \begin{cases} \pi b^{-1}(b) + (1-\pi) & if\ q = 1 \\ (1-\pi)b^{-1}(b) + \pi & if\ q = 0. \end{cases}$$

Since conditional on $q$, $(\theta_i, x_i)$ are drawn iid, it holds that the odds that buyer $i$ wins is $w(b^{-1}(b), \theta^*, q)^{n-1}$. By Bayes rule, it then implies that buyer $i$'s valuation, conditional on winning is

$$w_i = \theta_i \left[ \frac{\lambda \pi w(b^{-1}(b), \theta^*, 1)}{\lambda \pi w(b^{-1}(b), \theta^*, 1) + (1-\lambda)(1-\pi) w(b^{-1}(b), \theta^*, 0)} \right].$$

Note that for a strategy $b(\cdot)$ to form in equilibrium, it must be the case that the optimal bid is $b(\theta_i)$ and hence his valuation conditional on winning is

$$w_i = w(\theta_i) = \theta_i \left[ \frac{\lambda \pi w(\theta_i, \theta^*, 1)}{\lambda \pi w(\theta_i, \theta^*, 1) + (1-\lambda)(1-\pi) w(\theta_i, \theta^*, 0)} \right].$$

Next, let $\Phi(\cdot)$ denote the distribution of the second highest valuation private value conditional on receiving good news, then each buyer $i$'s payoff from bidding $b \in [p, b(1)]$ is

$$r(b) = \int_{w^{-1}(p)}^{b^{-1}(b)} [w(\theta_i) - \max\{p, b(y)\}] d\Phi(y).$$

This means that the optimal bid must satisfy the first order condition, which implies that

$$0 = r'(b) = \frac{w(\theta_i) - b(b^{-1}(b))]}{b'(b)]} = \frac{w(\theta_i) - b}{b'(b)]}. \tag{A1}$$

and in equilibrium $B = b(\theta_i)$. Hence, the equilibrium condition implies, as desired, that

$$b(\theta_i) = w(\theta_i).$$

This implies that the only strictly increasing bidding function that can be sustained in a symmetric, monotone equilibrium is to bid one's valuation conditional on winning. In the following section, I use this observation to derive the seller's problem.

### *A2. Optimal Reserve Price.*

I now state and solve the seller's problem. Rather than picking $p^*$ the seller might as well pick $\theta^*$ such that $p^* = w(\theta^*)$.

Suppose that the seller picks a value $\theta$ and $m \in \{0, 1, \cdots, n\}$ buyers observe good news. Then the seller expects to keep his item with odds $\theta^m$ and to net a payoff of

$$r(\theta, m) = m\theta(1-\theta)\theta^{n-1} + \int_\theta^1 m(m-1)x^{m-1}(1-x)dx + \theta^m \theta_s$$

Next, the odds of $m$ buyers receiving good news are $\tilde{\lambda}(m) = \binom{n}{m}[\lambda\pi^m(1-\pi)^{n-m} + (1-\lambda)(1-\pi)^m \pi^{n-m}]$. This implies that the seller's expected revenue from the choice of $\theta$ is the expected revenue of $r(\theta) = \tilde{\lambda}(m) r(\theta, m)$. Lastly, the seller's optimal choice solves $\max_{\theta \in [0,1]} r(\theta)$.

### *A3. Contracts, For Online Publication*

In this subsection, I present the terms of trade that the seller may offer buyers. The auction format is fixed, i.e. the protocol determining how buyers who participate in the auction interact with each other. The seller, however, picks who wins the good and how much the winner pays subject to the allocation rule being implementable by an equilibrium.

The resulting extensive form game satisfies the following conditions. First, if a buyer decides to not participate in the auction, then he neither wins the good nor makes a payment. On the other hand, if at least one player decides to participate in the auction, the item sells to whichever buyers holds the highest valuation at a price that is incentive compatible. Lastly, I assume that buyers are treated symmetrically, i.e. if two buyers have the same valuation, then they expect to win the auction with equal probabilities and to make the same expected payment.

Auction Format. — I assumed that the seller runs second-price auctions; is this necessary? My result holds if there exist negative selection among buyers choosing to participate in the auction. I now present a general auction setting in which my results persists. First, I model the auction procedure as a particular type of extensive form game that I call an "auction format". Next, I assume that the seller gets to pick a particular kind of indirect contract allowing that must form an equilibrium in the auction format.

I first define the set of outcomes. An outcome is either a buyer who wins the good and the payment he makes or a failure to trade, i.e. let the set of buyers be $\mathcal{I}$, then the set of outcomes is

(Set of Outcomes) $$\mathcal{X} \equiv \{(i, p_i) | i \in \mathcal{I}, p_i \in \Re\} \cup \{\text{failure}\}.$$

Note, the outcome $x =$ failure means that no buyer won the good. Next, I define the auction format.

DEFINITION 6 (Auction Format, is an extensive form game): *An auction format is a tuple* $\Gamma \equiv \{\mathcal{H}, \succeq, \rho, A, \mathcal{A}, (\mathfrak{I}_i)_{i\in\mathcal{I}}\}$ *consisting of*

*i. A game tree* $(\mathcal{H}, \succeq)$ *with initial history* $\hat{h}_o$*,* *ii.* *The set of terminal histories is* $\mathcal{Z}(\subset \mathcal{H})$ *such that* $\#\mathcal{X} = \#\mathcal{Z}$*,*

*iii.* *A function assigning buyers to non-terminal histories:* $\rho : \mathcal{H} - \mathcal{Z} \to 2^{\mathcal{I}} - \{\emptyset\}$*. This function denotes who get to take an action when.*

*iv.* *A set of acts* $A$ *and I assume that* $[0, 1] \subset A$*,*

*v.* *A map from non-terminal histories to acts feasible at the history in question:* $\mathcal{A} : \mathcal{H} - \mathcal{Z} \to 2^{A} - \{\emptyset\}$*,*

*vi.* *In the initial period, buyers have a participation decision: i.e.* $\rho(\hat{h}_o) = \mathcal{I}$ *and for each buyer* $i$*, it holds that* $\mathcal{A}(\hat{h}_o) = \{0, 1\}$ *such that if a buyer* $i$ *plays* $1$*, then they succeeding outcomes is not in* $\{i\} \cup \Re$ *and* $i$ *does not play in a subsequent non-terminal history.*

*vii.* *For each buyer* $i$*, there exists a collection of measurable information sets* $\mathfrak{I}_i$ *that are well defined, i.e. for each set* $B_i \in \mathfrak{I}_i$ *and pair of histories* $h, h' \in B_i$*, it holds that* $\mathcal{A}(h) = \mathcal{A}(h')$*.*

*viii.* *I assume that the game has perfect recall and a PBE.*

This definition allows for the second and first price as well as English auctions. Next, I define a strategy to the game above.

DEFINITION 7 (Behavioral Strategy): *A (behavioral) strategy for buyer* $i$ *is a measurable function* $\sigma_i : \mathcal{T} \times \mathfrak{I} \to \Delta(A)$ *such that for every set* $B_i \in \mathfrak{I}_i$ *and type* $\tau \in \mathcal{T}$*, it holds that* $supp \sigma_i(\tau, B_i) \subset \mathcal{A}(h_i)$ *for some history* $h_i \in B_i$*. Note that from henceforth, I denote the set of actions directly as a function of his information set.*

I can now define a PBE. Note that this is important since the seller picks an equilibrium of the game.

DEFINITION 8 (PBE): *Given a function* $g \equiv (q, p) : \mathcal{Z} \to \mathcal{X}$*, a PBE is a collection of behavioral functions* $(\sigma_i)$ *and beliefs such that for every set buyer* $i$*, set* $B_i \in \mathfrak{I}_i$ *and type* $\tau \in \mathcal{T}$*, it holds that* $\sigma_i(\tau_i, B_i)$ *solves*

$$u_i(\tau_i, B_i) = \max_{a \in \mathcal{B}_\rangle} E[u(\tau, x_{-i})|B_i,\ i \text{ wins}]E[q[a, \sigma_{-i}]|B_i] - E[p[a, \sigma_{-i}]|B_i] \tag{A2}$$

*where* $q(\cdot)$ *denotes the expected probability that buyer* $i$ *wins the good,* $p(\cdot)$ *the payment he makes, and the expectations are made given beliefs.*

The next subsection, I allow the seller to pick an outcome function $g(\cdot)$ given a fixed set of constraints. The seller implicitly picks an indirect mechanism that must form an equilibrium in the extensive form game in question.

Terms of Trade. — I now define a set of contracts that the seller can offer buyers. The seller announces a rule $g(\cdot)$ denoting who wins the good and how much the winner pays. Meanwhile, each buyer $i$ either reports a type $\tau_i$ or decides to not participate by reporting $\emptyset$. For what follows, assume that beliefs are given by $\beta = \{(F_i, K_i)\}$ where $F_i$ is CDF denoting each buyer $j \neq i$'s beliefs of $x_i$; whereas $K_i$ denote beliefs regarding the private value $\theta_i$. Next, define a buyer $i$'s valuation given beliefs $\beta$ and a type $\tau_i$ as

$$v(\tau_i, \beta) = E_\beta[u(\tau, x_{-i})|\tau_i].$$

I now define the terms of trade.

DEFINITION 9 (Terms of Trade): *Given beliefs $\beta = \{(F_i, K_i)\}$, the terms of trade are a triple $m \equiv (q, \pi, p)$ consisting of a minimal trading valuation $p \in [0,1]$ as well as an allocation and transfer rule pair $(q, \pi) : (\mathcal{T} \cup \{\emptyset\})^n \to [0,1]^n \times \Re^n$ such that for every collection of reports $\tilde{r} \equiv (r_j)$, it holds that*

*i.* *$q_i(\tilde{r}) \in [0,1]$ is the probability $i$ wins and $\pi_i(\tilde{r}) \in [p, \infty)$ $i$'s payment,*

*ii.* Buyers who abstain or have valuations below $p$ neither win or make transfers*: for every buyer $i$ report $r_i = \emptyset$ or $r_i = \tau_i$ such that $v(\tau_i, \beta) < p$, it holds that $q_i(\emptyset, \tilde{r}_{-i}) = \pi_i(\emptyset, \tilde{r}_{-i}) = 0$, $\forall\ \tilde{r}_{-i}$,*

*iii.* If a buyer reports a valuation above $p$, the good sells*: For every collection of reports $\tilde{r}$, if some buyer $i$ reports $r_i = \tau_i \in \tilde{r}$ such that $v(\tau_i, \beta) \geq p$, then $\sum_{i \in \mathcal{I}} q_i(\tilde{r}) = 1$,*

*iv.* The buyer with the highest valuation wins the good*: Suppose that there exists a report $r_i = \tau_i$ such that $v(\tau_i, \beta) \geq p$, then define the set*

$$W(\tilde{r}) = \{i \in \mathcal{I} : v(\tau_i, \beta) \geq v(\tau_j, \beta),\ or\ r_j = \emptyset \forall\ j \in \mathcal{I}\} \tag{A3}$$

*and for each buyer $i \in W(\tilde{r})$, let $q_i(\tilde{r}) = 1/\#W(\tilde{r})$,* *v.* The mechanism can be implemented*: there exists a strategy profile* $(\sigma_i)$ *and a function* $g(\cdot)$ *such that given* $g(\cdot)$, $(\sigma_i)$ *is a PBE and the composition of* $(\sigma_i)$ *and* $g$ *implements* $m$*,*

*vi.* The seller implements the mechanism in the revenue maximizing PBE*: There does not exists an alternative PBE* $(\sigma_i')$ *such that the expected revenue to the seller from implementing* $m$ *via* $(\sigma_i')$ *is strictly higher than implementing* $m$ *via PBE* $(\sigma_i)$*.*

I now explain this definition. Terms of trade are an indirect mechanism denoting who wins the good, how much each buyer pays, and who is excluded. They also satisfy the following conditions. First, If a buyer reports a type associate with a valuation above a cutoff $p$, the item sells. Second, the agent who values the good the most wins the auction. Note that this assumption, in general, implies that an auction like Myerson (1981) is not feasible. Next, I assume that the mechanism can be implemented via a mechanism in equilibrium and it maximizes the seller's revenues.

Note that the only lever controlled by the seller is the cutoff value $p$. The revenues as a function of $p$ when beliefs are $\beta = \{(F_i, K_i)\}$ can be expressed as a function of the virtual values of valuations, i.e. a distribution over $v(\tau, \beta)$ that is expected to have a CDF $H_i$ for each buyer $i$. Define the ironed out virtual value when beliefs over valuations are $H_i$ as $\bar{\phi}(\cdot, H_i)$ and the expected revenues is

$$r(p, (H_i)) = E_H[\max_i \chi(v_i \geq p)\bar{\phi}(v_i, H_i)\}] + \theta_s \prod_{i \in \mathcal{I}} H_i(p)$$

for $\chi(\cdot)$ is an indicator function.

I conclude this section noting that since the seller only manages the choice $p$ and incentives are increasing in valuation. Then, one can define a game where the seller still picks reserve prices $p_t$ and buyers pick rules to participate in the auction. The resulting games will have negative selection among buyers, so the results in this paper follow.

## Two period version of toy model: Equilibrium Characterization

This section derives the equilibrium analyzed in section IV. Since actions are perfectly observable and the game ends in period 1, all PBE are payoff equivalent to the equilibrium that is derived via Backward induction in pure strategies.

Period $t = 1$. — Suppose that the item remains unsold in period 1 and that agents expect that the buyers observed bad news (i.e. $x_i = 1$) or they observed good news but also private values below a cutoff value $\theta_1$. The seller first picks a reserve price $p_1 \geq \theta_s$ and then buyers can decide whether to participate.

Let us first characterize buyers' decision to participate in the auction. Note that buyers participating in the auction bid their valuation conditional on winning follows the same argument as in the static auction case. Since the seller cannot re-auction his good in any subsequent period, buyers follow a myopic participation rule, i.e. each buyer $i$ participates in the auction iff $b_{i1} \geq p_1$. Observe that $b_{i1}$ was derived in the main text of section IV.

I now characterize the seller's problem. The seller expects that for each reserve price $p_1 \in [\theta_1, 1]$, buyers with valuations conditional on winning that are greater than $p_1$ participate in the auction, i.e. each buyer $i$ bids iff $b_{i1} \geq p_1$. Furthermore, $b_{i1} = b_1(\theta_i)$ for a strictly increasing function $b_1(\cdot)$. This implies that the seller can might as well pick a private value $\theta_2 \in [\theta_s, \theta_1]$ such that $p_1 = b_1(\theta_2)$. The revenue from such choice is

$$r_1(\theta_2) = \int_{\theta_2}^{\theta_1} b_1(y) d\frac{\Phi(y)}{\Phi(\theta_1)} + [\lambda\pi + (1-\lambda)(1-\pi)] \left(\frac{\theta_2}{\theta_1}\right)^n \theta_s \tag{B1}$$

where $\Phi(\cdot)$ is the distribution of the second highest valuation among buyers who receives good news.The optimal choice solves

$$r_1(\theta_1) = \max_{\theta_2 \in [0,\theta_1]} r_1(\theta_2, \theta_1).$$

Period $t = 0$.. — I now characterize the equilibrium in period 0. Assume that the seller picks a reserve price $p_0 \geq \theta_s$. I claim that there exists a cutoff value of $\theta_1 \in [\theta_s, 1]$ such that only buyers observing good news and a private value above $\theta_1$ bid their valuation conditional on winning.

Buyers expect that the item may be re-auctioned in period 1 if the item fails to sell. Each buyer $i$ expects that if they forgo bidding in period 0, they can play in the following period. If buyer $i$ is further indifferent, he expects to win iff he is the sole bidder and his payoff is $b_0(\theta_1) - p_0$. Meanwhile, if buyer $i$ waits, then he expects to win in the subsequent history with probability 1 and to pay the second highest valuation in that period or the subsequent reserve price is he is the sole bidder. Buyer $i$'s indifference hence equals to

$$b_0(\theta_1) - p_0 = \delta\Big[b(\theta_1,\theta_1) - p_1[\theta_1]\{\lambda_1 w(\theta_1,\theta_2(\theta_1),1)^{n-1} + (1-\lambda)w(\theta_1,\theta_2(\theta_1),0)^{n-1}\} - \int_{\theta_2}^{\theta_1} b(y,\theta_1)dF_2(y,\theta_1)\Big] \tag{B2}$$

where $\theta_2(\cdot), p_1(\cdot)$ are the seller's policy functions in period 1 as a function of each choice $\theta_1 \in [\theta_s, 1]$, $F_2(\cdot,\theta_1)$ is the CDF given for the second higher private value given that all buyers observing good news observed private values below $\theta_1$, let $F(\cdot,\theta_1)$ be the distribution for a given buyer, and

$$b(\theta,\theta_i) \equiv \theta E[q|x_i = 1, i \text{ wins}, \forall j \neq i \text{ s.t.} x_j = 1, \theta_j \leq \theta_1].$$

I now study the seller's problem. The seller might as well pick $\theta_1$ since equation B2 allows me to derive a reserve price $p_0 = p_0(\theta_1)$. The seller expects to either net the revenues from the appropriate, second-price auction or to not sell and net the expected, discounted revenues from an offering in the subsequent auction. Hence, the seller faces the following problem

$$r_0 = \max_{\theta_1\in[0,1]} p_0(\theta_1)[1-F(y,1)]F(y,1)^{n-1} + \int_{\theta_1}^{1}\int_{\theta_1}^{x} b_0(y)dF_2(y,1)dF_1(x,1) + \delta r_1(\theta_1) \tag{B3}$$

where the function $F_1(\cdot,1)$ denotes the distribution of the highest valuation. Next, the objective function is continuous and $[0,1]$ is compact. Therefore, there extreme value theorem implies that there exists a solution to equation B3, an equilibrium is well defined, and there exists a value $\theta_1$ as discussed in the main text.

## Durable Goods Market, For Online Publication

Up to this point, I showed that interdependence precludes the Coase conjecture in auction settings, but doe this insight persists in non-auction settings? The answer is yes. I present a durable goods monopoly example in which interdependence allows the seller to contravene the Coase conjecture in a stationary equilibrium.

A monopolist offers a durable good to a unit mass of consumers. Nature first draws a common quality $q$ such that $\ln q \sim N(\mu,\sigma)$ for $(\mu,\bar{\sigma}) \gg 0$. Then nature privately informs each buyer $i$ a private signal $x_i = q + \epsilon_i$ for $\epsilon_i \sim N(0,\hat{\sigma}), \hat{\sigma} > 0$, such that for each pair of distinct buyers $i, j$, it holds that $\epsilon_i$ is pairwise independent of $\epsilon_j$. At each period $t = 0, 1, \ldots$, the seller first announces a price $p_t \geq 0$. Buyers then decide the either purchase the good (and exit the market) or wait. If a buyer purchases the good at a period $t$, at a price of $p_t$, then his payoff is $\delta^t(q - p_t)$ for some common $\delta \in (0, 1)$.

I now define histories, strategies, and equilibrium. At each period $t$ a history consists of the set of past prices and share of buyers who purchased the good: i.e. $h_t = \{p_s, m_s\}_{s=0}^{t-\Delta} \in H_t \equiv \Re_+^t \times [0,1]^t$. Next, a seller strategy is a collection of functions $(p_t), \forall t, p_t : H_t \to \Re_+$ denoting the current price; meanwhile, I an anonymous buyer strategy is a collection of functions $(c_t), \forall t, c_t : H_t \times \Re \times \Re_+ \to [0,1]$ where for each tuple $(h_t, x_i, p_t)$, $c_t(h_t, x_i, p_t)$ denotes the probability that the buyer purchases the good conditional on not previously purchasing the item. A PBE is then a pair $\beta = \{(p_t), (c_t)\}$ coupled with beliefs such that given beliefs, the strategies are sequentially rational, beliefs are derived from $\beta$ via Bayes rule whenever possible.

I claim that there exists an equilibrium where the seller fixes an initial price at $p_0 > 0$ and for every period $t > 0$ he fixes prices at $p_t = q$. First, buyers expecting this seller strategy profile to be played in an equilibrium expect that there are no gains from trade to be had by delaying their purchases decide to buy the good at a price of $p_0$ provided that $p_0 \leq E[q|x_i] = e^{x_i}$. Hence, for each quality $q$, the share of buyers purchasing the good in period 0 if $D(p_0, q) = 1 - \Phi\left[\frac{1}{\hat{\sigma}} \ln\left(\frac{p_0}{q}\right)\right]$ for $\Phi(.)$ being the CDF of the standard normal distribution. Observe that the corresponding p.d.f. of a normal distribution with mean $\mu$ ans variance $\sigma$ will be defined as $\phi(x, \mu, \sigma)$.

The seller also conjectures that he would fix prices after period $\Delta$ and expects that for each price $p_0$ he picks, his revenues are

$$\begin{aligned} r(p_0) &= \int_0^\infty p_0\left\{1 - \Phi\left[\frac{1}{\hat{\sigma}} \ln\left(\frac{p_0}{q}\right)\right]\right\} + \delta q \Phi\left[\frac{1}{\hat{\sigma}} \ln\left(\frac{p_0}{q}\right)\right] d\phi(\ln q, \mu, \sigma) \\ &= p_0 + \int_0^\infty \phi(\ln q, \mu, \sigma) \Phi\left[\frac{1}{\hat{\sigma}} \ln\left(\frac{p_0}{q}\right)\right] [\delta q - p_0] dq. \end{aligned}$$

The optimal price $p_0$ then maximizes $r(p_0)$ among all prices $p_0 \geq 0$ and it satisfies the first order condition

$$\text{(C1)} \quad \hat{\sigma} + \int_0^\infty \phi(\ln q, \mu, \sigma) \phi\left[\ln\left(\frac{p_0}{q}\right), 0, \hat{\sigma}\right]\left[\delta - \frac{p_0}{q}\right] dq = \hat{\sigma} \int_0^\infty \phi(\ln q, \mu, \sigma) \Phi\left[\frac{1}{\hat{\sigma}} \ln\left(\frac{p_0}{q}\right)\right] dq.$$

Now, once buyers purchase the good in period 0, they all observe a share of buyers purchasing the good $m_0 \in (0, q)$ and this quantity can only be associated with a unique quality $q$ satisfying $D(p_0, q) = m_0$. Hence, buyers learn that the common value equals to $q$. The seller then fixes the price at $q$ and buyers are indifferent between buying and waiting, so it is an equilibrium for them to buy the good.

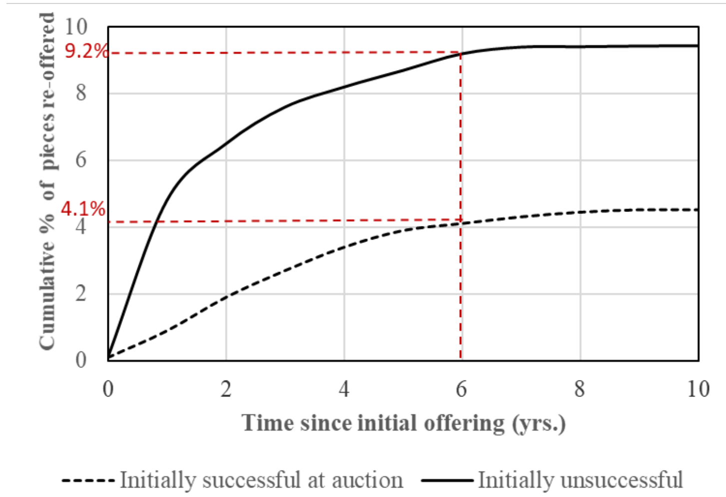
## Empirical Evidence

I focused on the theoretical observation that interdependence in valuations prevents the Coase conjecture in auctions. Art auctions are an empirical analog to the setting discussed and, in this appendix, I study the pattern of sequential auctioning of paintings that previously failed to sell. It is already known that if an artwork is sent to auction, it fails to sell, and it is successfully re-auctioned at the same auction house and location, then the expected sale price is lower than would have been expected from a comparable artwork that was not previously auctioned unsuccessfully. This is known as *value burning* in the literature. The model, however, suggest that artworks may not be re-auctioned, but it is not clear whether the seller avoids re-offering artworks altogether. I analyze the frequency at which artworks that fail to sell are re-auctioned later using data from Beggs and Grady (2009). My main result is that sellers seldom re-auction artworks at public auctions once the artwork sells.

I will first describe the data at hand. The data consists of Impressionist art auctions held by Sotheby's and Christie's from 1980 to 1991 in London and New York. The art market, in general, has been concentrated in these two cities since the mid-1940s. ArtBasel (2020) shows that, even to this day, Christie's and Sotheby's of New York and London account for more than sixty percent of all sales by volume and revenue and this share has been roughly constant for the preceding decade. Nonetheless, it is possible for an artwork to have failed to sell at an auction house but being re-auctioned at a different location that is not in the sample. For example, a Monet may fail to sell in Sotheby's London, fails to sell, and it is later re-auctioned by Phillips in Paris. Next, it is difficult to extend the dataset to a more recent period since auction houses do not publish their failed sales and one would have to include offerings in Hong Kong.

I first note that around thirty percent of Impressionist artworks fail to sell. Figure DII shows that the share of artworks that failed to sell varied between eighteen and forty percent in the analysis period. This rate is in line with previous studies pertaining the Impressionist

Figure DI. : *Share of artworks re-offered by the time since last auction.*




(a) Source. *Author's calculation using Beggs and Grady (2009) data.*

art market. Furthermore, Ashenfelter and Grady (2003) and Ashenfelter (1989) show that the share of items brought to auction that fail to sell does vary across art movements.

I then estimate that roughly ten percent of artworks that failed to sell were eventually re-auctioned. Meanwhile, around four percent of artworks that initially sold at auction are also re-auctioned. This implies that failing to sell is associated with roughly a five percentage points increase in the likelihood that an item is re-offered.

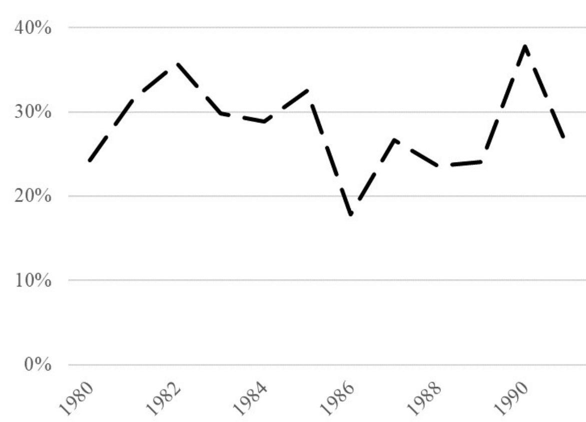



Figure DII. : *Share of Impressionist paintings that failed to sell at auction.*

Figure DIa further plots the cumulative share of artworks re-auctioned as a function of the time since the initial offering. This figure allows us to observe whether artworks that failed to sell are, on average, re-auctioned sooner than artworks that were successfully sold. Roughly 60 percent of all paintings that initially failed to sell and were re-auctioned within the first two years of the initial offering. But the rest are sold after a considerably longer wait. This implies that the average time between offerings of these artworks is approximately five and a half years. Meanwhile, roughly half of the artworks that did sell at auctioned and re-auctioned at later dates.

Next, I consider whether these quantitative observations are a result of confounding factors. For example, artworks that failed to sell might have been offered at auctions with fewer buyers, created by less recognizable artist, or had other properties which were pathologically unappealing to their art market in the 1980s. Controlling for market, artist, price estimates and artwork fixed effects, I find that an item failing to sell is associated with a 5.2 percentage point increase in the probabilities that the item is re-offered as reported in table DI. On the other hand, I find that an item failing to sell is associate with a 0.04 year *increase* in the time between re-offerings. Such difference is nonetheless neither economically nor statistically significant. As usual, I make no claims that these coefficients reflect any sort of causal relation nor assume that there could be informative of any kind of policy experiment. What I claim is that these correlates are informative of descriptive, empirical patterns of interest. In other words, had the Coase conjecture better described reality and the seller would prefers frequently auctioning his good, it should be a near certainty that goods that fail to sell are quickly and often re-auctioned: and not 5 percentage points more often than paintings that sold and after the shortest wait possible and not 5 years.

These results suggest that an item is slightly more likely to be re-offered if it failed to sell but that there is limited evidence to support that these artworks were re-auctioned sooner than their counterparts. Thus, there is little empirical evidence supporting the belief that dynamic commitment considerations are empirically relevant for the market in consideration.

I conclude this empirical section making several caveats. First, extending to the current era is difficult. This is because auction houses do not freely publish reports on the artworks that failed to sell. Hong Kong also emerged as a significant art market, so expanding the dataset would require compiling data from China as well. Next, I do not make any claims that these patterns must extend to other art markets or to auctions in general. Further empirical work should clarify this point further. The last point is that sellers have non-auction avenues to sell their artworks (e.g., private sales and direct offerings to collectors) and getting information on

| **Variable** | *Item re-offered* | | *Wait* |
|---|---|---|---|
| *Failed to sell* | 0.052*** | | 0.036 |
| *Auctioned in New York* | 0.011** | | -0.439*** |
| *Auctioned by Sotheby's* | -0.017*** | | -0.193*** |
| *+Year, artist, and art style fixed effects* | | | |
| *Sample Size* | 7,543 | | 4,666 |
| *Pseudo R-square* | 0.035 | *R-Square* | 0.631 |

Table DI: Marginal effect of no sale the probability that the item is re-auctioned within ten years of its last offering and the expected wait on being re-auctioned among artworks auction multiple times.

those markets is notoriously difficult.

## Proofs.

This section of the paper presents the proofs. First, I present the proof pertaining the motivating example. I then present the proofs to the auxiliary results. Lastly, I present the proof of the main theorem.

*Proof of lemma 2.*

**Proof**

The proof proceeds as follows. I will posit that the seller posts the optimal strategy and that buyers expect that the good will not be re-offered. Next, I rationalize the conjecture posited.

Suppose that the seller posts a price schedule $p_t = p^*$ for each period $t$ and the item fails to sell in period 0. Then, every buyer who observed good news has a valuation equal to $v_{i1} \equiv \theta_i q(\theta^*) \leq \theta^* q(\theta^*)$ and

$$\begin{aligned}\theta^* E[q|x_i=1, agent\ w/\ \theta_i=\theta^*,\ wins] &= \frac{\theta_* \pi\lambda w(\theta^*,1)^{n-1}}{\pi\lambda w(\theta^*,1)^{n-1}+(1-\pi)(1-\lambda)w(\theta^*,0)^{n-1}} \\ &= \frac{\theta^*\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1}}{\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1}+1} \\ &\leq \frac{\frac{\theta^*\theta_s}{\theta^*-\theta_s}}{\frac{\theta_s}{\theta^*-\theta_s}+1} \\ &= \frac{\theta_s\theta^*}{\theta_s+(\theta^*-\theta_s)} = \theta_s. \end{aligned} \tag{E1}$$

Observe that the second line uses the condition stated in the prompt. Next, notice that no buyer is willing to bid more than $\theta_s$ in period 0, so the seller prefers keeping the item and when he posts $p_1 = p^*$ no buyer bids with probability 1.

Assume that by period $t \geq 1$, the seller maintains the prices at $p_s = p^*$ for $s \leq t$. Then in period $t$, no buyer is expected to submit a bid at any period $s \geq 1$ and thus beliefs do not change from period 1. Consequently, all buyers still value the good less than $\theta_s$, the seller still prefers keeping the good, and he might as well fix prices at $p_t = p^*$.

The opposite direction is immediate. Suppose that $\theta^* q(\theta^*) \leq \theta_s$, then

$$\begin{aligned}\theta_s \geq \theta^* q(\theta^*) &= \frac{\theta_* \pi\lambda w(\theta^*,1)^{n-1}}{\pi\lambda w(\theta^*,1)^{n-1}+(1-\pi)(1-\lambda)w(\theta^*,0)^{n-1}} \\ &= \frac{\theta^*\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1}}{\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1}+1}. \end{aligned} \tag{E2}$$

If one re-arranges the inequality at hand, it holds

$$\theta_s\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1}+\theta_s \geq \theta^*\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{\pi}{1-\pi}\right)\left[\frac{w[\theta^*,1]}{w[\theta^*,0]}\right]^{n-1} \tag{E3}$$

and collecting terms leads to equation 2. □

### *E1. Proof of Lemma 3, Theorem 1, and Theorem 2.*

I now prove the auxiliary results. First, I prove that buyers follow a threshold strategy in every equilibrium. Next, I establish of progressive pessimism. The last prove provides bounds on the equilibrium revenues that the seller attains.

Buyers follow a threshold strategy, lemma 3.. — **Proof**

I first prove that buyers follow a threshold strategy $(u_t)$. This argument has two steps. First, I show that if buyer $i$ has a greater valuation at some period $t$ when his type is $\tau$ than when his type if $\tau'$, then his valuation given type $\tau$ remains greater than given type $\tau'$ for every period $s \geq t$. The next result establishes that the net payoff between bidding right away and waiting is non-decreasing in the current valuation.

Single Crossing Property Holds. — The first section of the proof first establishes a non-crossing difference in payoffs.

PROPOSITION 6: *For every pair* $\tau, \tau' \in [0,1]^2$ *and* $x_{-i}, \in [0,1]^{n-1}$, $\Delta(\tau, \tau', x_{-i}) \equiv u(\tau, x_{-i}) - u(\tau', x_{-i}) \geq 0$ *iff* $\Delta(\tau, \tau', 0) \geq 0$.

**Proof**

Fix some pair $\tau, \tau' \in [0,1]^2$ and a $x_{-i} \in [0,1]^{n-1}$. First, if $\Delta(\tau, \tau', 0) \geq 0$, then the non-decreasing differences condition implies that $\Delta(\tau, \tau', x_{-i}) \geq 0$ for each $x_{-i} \in [0,1]^{n-1}$. This establishes the inverse direction. Next, suppose for contradiction that $\Delta(\tau, \ tau', x_{-i}) \geq 0$ but $\Delta(\tau, \tau', 0) < 0$. Since $u(.)$ is continuous, then $\Delta(\tau, \tau', .)$ is also continuous. Since $[0,1]^{n-1}$ is connected, then the intermediate value theorem implies that $\Delta(\tau, \tau', [0,1]^{n-1})$ is also connected. Hence, there exists some $x'_{-i} \in [0,1]^{n-1}$ such that $\Delta(\tau, \tau', x'_{-i}) = 0$ and by non-decreasing difference, it holds that

$$0 < |\Delta(\tau, \tau', 0)| \leq |\Delta(\tau, \tau', x'_{-i})| \leq \Delta(\tau, \tau', x'_{-i}) = 0. \tag{E4}$$

This is a contradiction and concludes the proof. □

Belief Independence Ordering in Valuations.. — Next, I establish that if buyer $i$ values the good more when he observes $\tau$ than when he observes $\tau'$ for some given beliefs regarding $x_{-i}$. Then he would still value the good more when his type is $\tau$ rather $\tau'$ given any other alternative belief regarding $x_{-i}$.

COROLLARY 7: *Fix some PBE, history $h$, and suppose that for some pair of types $\tau, \tau'$ it holds that $E[u(\tau, x_{-i})|h] \geq E[u(\tau', x_{-i})|h]$, then for every $h'$, it holds that $E[u(\tau, x_{-i})|h'] \geq E[u(\tau', x_{-i})|h']$.*

**Proof** Fix some PBE, history $h$, pair of types $\tau, \tau'$, and assume that $E[u(\tau, x_{-i})|h] \geq E[u(\tau', x_{-i})|h]$. This equivalently implies that $E[\Delta(\tau, \tau', x_{-i})|h] \geq 0$ and hence there exists some $x_{-i} \in [0, 1]^{n-1}$ such that $\Delta(\tau, \tau', x_{-i}) \geq 0$. By proposition 6, it follows that for each $x'_{-i}$ ,$\Delta(\tau, \tau', x'_{-i}) \geq 0$. Consequently, for each history $h'$

$$0 = E[0|h'] \leq E[\Delta(\tau, \tau', x'_{-i})|h']. \tag{E5}$$

This concludes the proof. □

Payoffs from participating in an auction increases with valuations. — I now prove that the payoff that a buyer receives from participating in an auction is non-decreasing in his valuation conditional on winning.

PROPOSITION 8: *For every feasible terms of trade $m = (p, x, t)$ and belief on valuations $H$, buyer payoffs from participating in the terms of trade is non-decreasing in his valuation $v$. Furthermore if some buyer with a valuations $v$ wins the item with a strictly positive probability, then the*

**Proof** Fix some reserve price $p$, a belief $H$, and a pair of valuations conditional on winning $v, v'$ such that $p \leq v \leq v' \leq 1$. Then, the payoff that a buyer nets equals to his valuation minus the second highest valuation

$$\begin{aligned} V(v', H) &= \int_0^{v'} (v' - \max\{y, p\}) dH_2(y) \\ &= \int_0^{v} (v' - \max\{y, p\}) dH_2(y) + \int_v^{v'} (v' - \max\{y, p\}) dH_2(y) \\ &\geq \int_0^{v} (v - \max\{y, p\}) dH_2(y) + \int_v^{v'} (v' - \max\{y, p\}) dH_2(y) \\ &= V(v', H) + \int_v^{v'} (v' - \max\{y, p\}) dH_2(y) \\ &\geq V(v', H) \end{aligned} \tag{E6}$$

where $H_2(\cdot)$ refers to the distribution of the second highest valuation given belief CDF $H$ and $V(x, H)$ is the valuation that a buyer with a valuation conditional on winning the good when beliefs are given by $H$. This concludes the proof. $\square$

Buyers follow a Threshold strategy. — I now prove that the decision to participate in the auction must be characterized by a threshold strategy. The argument presented here is standard.

Fix some PBE, period $t$, history $h_t$, and a buyer $i$. Assume that buyer $i$ has a type $\tau$ and he weakly prefers to participate in the current terms of trade $m_t$. Then it must be the case that his payoff from participating in the current terms of trade is weakly greater than waiting: formally, let the buyer's valuation be $v(\tau, h_t) \equiv E[u(\tau, x_{-i})|h_t]$

$$V(v(\tau, h_t), H(.|h_\tau)) \geq W(\tau, h_t) \equiv E\left[\sum_{s=1}^{\infty} \delta^\tau b_{t+s}(\tau, h_{t+s}) s_{t+s}(h_{t+s}) V(v(\tau, h_{t+s}), H(.|h_{t+s}))|h_t\right] \geq 0 \tag{E7}$$

for $b_{t+s}(x, h_t)$ is the probability that a buyer participates in an auction conditional on the good remaining unsold by period $t+s$, the buyer choosing not to bid right away, and history $h_t$, $s_{t+s}(h_{t+s})$ denotes the probabilities that the item has failed to sell and $W(\tau, h_t)$ his option value from waiting. Now suppose that the buyer had a valuation $\tau'$ such that $V(v(\tau', h_t), H(.|h_t)) > V(v(\tau, h_t), H(.|h_t))$. I claim that if buyer had type $\tau'$ rather than type $\tau$, the seller still prefers bidding. First, observe that the type is unverifiable, so buyer $i$ observing $\tau'$ can replicate the equilibrium strategy he is supposed to pick when his type is $\tau$ instead, and vice versa, and cannot receive strictly higher payoff from deviating. Furthermore, as $v(\tau', h_t) \geq v(\tau, h_t)$ then corollary 7 implies that his valuation in any subsequent history $h_{t+s}$ satisfies that $v(\tau', h_{t+s}) \geq v(\tau, h_{t+s}$; moreover, proposition 8 implies that

$$V(v(\tau, h_{t+s}), H(.|h_{t+s})) \leq V(v(t, h_{s+\tau}), H(.|h_{s+\tau})). \tag{E8}$$

Therefore, it holds that the PBE equilibrium payoffs satisfy

$$U(\tau', h_t) \geq E\left[\sum_{\tau=0}^{\infty} \delta^\tau s_{s+\tau}(h_{s+\tau}) V(v(t', h_{s+\tau}), H(.|h_{s+\tau}))|h_t\right] \geq W(t, h_t) \tag{E9}$$

as the buyer with type $\tau$ must henceforth fix his participation to 1 and

$$U(\tau, h_t) \geq E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau}) b_{s+\tau}(t', h_{s+\tau}) V(v(t, h_{s+\tau}), H(.|h_{s+\tau}))|h_t\Big]. \tag{E10}$$

Consequently, the difference in equilibrium payoffs must satisfy

$$\begin{aligned} & U(\tau', h_t) - U(\tau, h_t) \geq \\ E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau}) & [V(v(t', h_{s+\tau}), H(.|h_{s+\tau})) - V(v(t, h_{s+\tau}), H(.|h_{s+\tau})]|h_t\Big] \geq 0. \end{aligned} \tag{E11}$$

However, since types are unverifiable, then the difference in options values from delay must be greater than the payoff that a buyer observing type $\tau$ could get if he plays the strategy of a buyer observing $\tau'$, i.e.

$$\begin{aligned} & W(t', h_t) - W(t, h_t) \leq \\ E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau}) b_{s+\tau}(t', h_{s+\tau}) & \Big\{V(v(t', h_{s+\tau}), H(.|h_{s+\tau})) - V(v(t, h_{s+\tau}), H(.|h_{s+\tau}))\Big\}|h_t\Big] \end{aligned} \tag{E12}$$

This implies that that

$$E\Big[\sum_{\tau=0}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau})[b_{s+\tau}(t', h_{s+\tau}) - 1]\Big\{V(v(t', h_{s+\tau}), H(.|h_{s+\tau})) - V(v(t, h_{s+\tau}), H(.|h_{s+\tau}))\Big\}|h_t\Big] \geq 0. \tag{E13}$$

But since for each history $h_{t+\tau}$, $V(v(\tau', h_{t+\tau}), H(.|h_{t+\tau})) \geq V(v(\tau, h_{t+\tau}), H(.|h_{t+\tau}))$ and the choices $1 \geq b_{t+s}(., \tau)$ and non-decreasing in the history set order, then it must hold that

$$E\Big[\sum_{\tau=1}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau})[b_{s+\tau}(t', h_{s+\tau}) - 1]\Big\{V(v(t', h_{s+\tau}), H(.|h_{s+\tau})) - V(v(t, h_{s+\tau}), H(.|h_{s+\tau}))\Big\}|h_t\Big] \leq 0 \tag{E14}$$

and

$$\begin{aligned} & [b_s(t', h_s) - 1]\{V(v(t', h_s), H(.|h_{s+})) - V(v(t, h_s), H(.|h_{s+\tau}))\} \\ +E\Big[\sum_{\tau=1}^{\infty} \delta^{\tau} s_{s+\tau}(h_{s+\tau})[b_{s+\tau}(t', h_{s+\tau}) - 1] & \Big\{V(v(t', h_{s+\tau}), H(.|h_{s+\tau})) - V(v(t, h_{s+\tau}), H(.|h_{s+\tau}))\Big\}|h_t\Big] = 0 \end{aligned} \tag{E15}$$

This concludes the proof since for the only way both equations hold is for the buyer observing history $\tau'$ to also bids immediately. $\square$

Progressive Pessimism, Theorem 1.. — **Proof**

This proof establishes that in every period $t$ and history $h_t$, prior beliefs $F_t(\cdot|h_t)$ likelihood ratio dominate posterior beliefs $F_t^{'}(\cdot|h_t)$. I present the argument for interdependent values since the argument for valuations and private values is identical to the one in question. It is worth noting that these proofs go by induction and that I present the induction step first and the initial condition.

Now that this lemma is established, it will be immediate that if $H_t[0, v_t]$ likelihood ratio dominated $H_{t+1}[0, u_{t+1}]$, for $u_{t+1} \leq v_t$, then $H_t/H_t(u_{t+1})[0, v_t]$ also likelihood ratio dominates $H_{t+1}[0, u_{t+1}]$.

Learning and Induction Step. — Fix some PBE whose threshold strategy is $(u_t)$, period $t$, history $h_{t+1}$, and assume that the prior belief are given by a CDF $F(.|h_t)$ denote each buyer $i$'s belief regarding each buyer $j \neq i$'s interdependent value $x_j$ with pdf $f_t(.|h_t)$, and the item remains unsold. Since the item remains unsold it holds that for every buyer $i$ $v(\tau_i, h_t) \leq u_t(h_{t+1})$. Therefore, for each interdependent value $x_j \in [0, 1]$ Bayes' rule implies that the posterior beliefs in period $t$, i.e. beliefs once the good fails to sell, satisfiy

$$\begin{aligned} f_{t+1}(x_j|h_{t+1}) &= \frac{Pr(v(x_j, \tilde{\theta}_j, h_t) \leq u_t(h_{t+1})|h_t)f(x_j|h_t)}{\int_0^1 Pr[v(y, \tilde{\theta}_j, h_t) \leq u_t(h_{t+1})|h_t]f(y|h_t)dy} \\ &= \frac{f(x_j|h_t)\int_{v(x_j,\tilde{\theta}_j,h_t)\leq u_t(h_{t+1})} K_t(\tilde{\theta}|h_t)d\tilde{\theta}}{\int_0^1 f(y|h_t)\int_{v(x'_j,\tilde{\theta}_j,h_t)\leq u_t(h_{t+1})} K_t(\tilde{\theta}'|h_t)d\tilde{\theta}'_j dy} \end{aligned} \tag{E16}$$

where $K_t(\cdot|h_t)$ are the equilibrium beliefs regarding private values, i.e. each agent $i$ in period $t$ and history $h_t$ expects that the private value of each peer $j$ is distributed given $\theta_j \sim K_t(\cdot|h_t)[0, 1]$.

Now, pick some pair $x_j, x'_j \in [0, 1]$ such that $x_j \leq x'_j$, then as $u(.)$ is strictly increasing in $(\theta, x)$, then so is $v(., h_t) = E[u(., x_{-i})|h_t, (\cdot), i \text{ wins}]$ for each history $h_t$ given that the expectation operator preserves monotonicity. By proposition 8, it further holds that if $(\theta, x_j) < (\theta, x'_j)$, then $v(\theta, x_j, h_t) \leq v(\theta, x'_j, h_t)$. This is because the payoff that each buyer would net from participating in an auction as well as the equilibrium payoff must be non-decreasing in each buyer's valuation. Consequently, for each pair of interdependent values $x_j, x'_j$ such that $x_j \geq x'_j$ the set of private values $\theta_j$ for which buyer $j$ does not participate in the auction when he observes $x_j$ is smaller than comparable set given $x'_j$:

$$\{\theta \in [0,1] | w(\theta, x'_j, h_t) \leq u_t(h_{t+1})\} \subset \{\theta \in [0,1] | w(\theta, x_j, h_t) \leq u_t(h_{t+1})\}. \tag{E17}$$

Since beliefs denote a measure on the set of private values and probability measures are monotone, then

$$Pr_t(V(x'_j, \tilde{\theta}_j, h_t) \leq \theta_s(h_{t+1}) | h_t) \leq Pr_t(V(x_j, \tilde{\theta}_j, h_t) \leq \theta_s(h_{t+1}) | h_t). \tag{E18}$$

Informally, this argument states that since payoffs are strictly increasing and $x'_j \leq x_j$, then there exists a larger collection of private values that buyer $j$ could have observed and his valuation to lie below any given cutoff. Next, if one divides $f_{t+1}(x'_j|h_{t+1})$ by $f_{t+1}(x_j|h_{t+1})$, then it holds that

$$\frac{f_{t+1}(x'_j|h_{t+1})}{f_{t+1}(x_j|h_{t+1})} = \left[\frac{f_t(x'_j|h_t)}{f_t(x_j|h_t)}\right]\left[\frac{Pr_t(v(x'_j, \tilde{\theta}_j, h_t) \leq u_t(h_{t+1})|h_t)}{Pr_t(v(x_j, \tilde{\theta}_j, h_t) \leq u_t(h_{t+1})|h_t)}\right] \leq \frac{f_t(x'_j|h_t)}{f_t(x_j|h_t)}. \tag{E19}$$

Since $x_j, x'_j$ were arbitrarily chosen, one can conclude that $F_t(.|h_t)$ likelihood ratio dominates $F_{t+1}(.|h_{t+1})$.

Initial Step. — In this part of the proof, I establish the initial step in the inductive argument. Fix some price $p_0 \in [0,1]$ and let $h_1 = \{p_0\}$. Then suppose that the item failed to sell, the each buyer $i$ updates his belief that his peer $j \neq i$ observes $x_j$ via Bayes rule as follows

$$f_0(x_j|h_1) = \frac{Pr(v_0(x_j, \tilde{\theta}_j) \leq u_0(h_1)) f(x_j)}{\int_0^1 Pr(v_0(y, \tilde{\theta}_j) \leq u_0(h_1)) f(y) dy} = \frac{f(x_j) \int_{v_0(x_j, \tilde{\theta}_j) \leq u_0(h_1)} k(\theta) d\theta}{\int_0^1 f(y) \int_{v_0(x'_j, \tilde{\theta}_j) \leq u_0(h_1)} K(\theta', y) d\theta' dy}. \tag{E20}$$

Now, pick some pair of interdependent values $x_j, x'_j$ such that $x_j < x'_j$, then the same argument in period $t$ and $h_t$ implies that

$$\{\theta \in [0,1] | v_0(\theta, x'_j) \leq u_0(h_1)\} \subset \{\theta \in [0,1] | v_0(\theta, x_j) \leq u_0(h_1)\}. \tag{E21}$$

Consequently, by the monotonicity of probability measures, it holds that

$$\text{(E22)} \qquad Pr(v_0(x_j', \tilde{\theta}_j) \leq u_0(h_1)|h_0) \leq Pr(v_0(x_j, \tilde{\theta}_j) \leq u_0(h_1)|h_0).$$

Lastly, if one divides $f_0'(x_j'|h_1)$ by $f_0'(x_j|h_1)$, it holds that

$$\text{(E23)} \qquad \frac{f_0(x_j'|h_1)}{f_0(x_j|h_1)} = \left[\frac{f(x_j')}{f(x_j)}\right]\left[\frac{Pr(v_0(x_j', \tilde{\theta}_j) \leq u_0(h_1))}{Pr(v_0(x_j, \tilde{\theta}_j) \leq u_0(h_1))}\right] \leq \frac{f(x_j')}{f(x_j)}.$$

Since the choice of $x_j, x_j' \in [0,1]$ and $p_0$ are arbitrary, then $F(\cdot)$ likelihood ratio dominates the posterior $F_0(\cdot|h_1)$ for each price $p_0$ such that $h_1 = \{p_0\}$. □

Proof of Corollary 4.. — This subsection characterizes how the dispersion in valuations falls over time.

**Proof**

I will first prove that valuations fall. Fix some PBE, period $t$, history $h_t$, and price $p_t$. Define $h_{t+1} = (h_t, p_t)$, then progressive pessimism, i.e. the previous theorem, states that $F_t(\cdot|h_t)$ likelihood ratio dominates $F_{t+1}(\cdot|h_{t+1})$. Next, progressive pessimism implies First order stochastic dominance. Consequently, since $u(\cdot)$ is strictly increasing, then for each type $\tau_i$, it holds that

$$\text{(E24)} \qquad v_t(\tau_i, h_t) = E_t[u(\tau_i, x_{-i})|h_t, \tau_i, i \text{ wins}] \geq E_{t+1}[u(\tau_i, x_{-i})|h_{t+1}, \tau_i, i \text{ wins}] = v_{t+1}(\tau_i, h_{t+1}).$$

Next, I show that the dispersion in valuations falls. Fix some PBE, some period $t$, a history $h_t$, price $p_t$, and a pair of types $\tau, \tau'$. Then let us suppress conditioning on winning the auction and it holds that

(E25)

$$\begin{aligned}
E_t[|v_t(\tau) - v_t(\tau')||h_t] &= E_t[|E_t[u(\tau, x_{-i}) - u(\tau, x_{-i})|h_t]||h_t] \\
&= E_t[|E_t[\Delta(\tau, \tau', x_{-i})|h_t]||h_t] \\
&= E_t[E_t[\Delta(\tau, \tau', x_{-i})|h_t]\chi(\Delta(\tau, \tau', \mathbf{0}) \geq 0) - [\Delta(\tau, \tau', x_{-i})|h_t]\chi(\Delta(\tau, \tau', \mathbf{0}) \leq 0)|h_t] \\
&\geq E_{t+1}[E_t[\Delta(\tau, \tau', x_{-i})|h_t]\chi(\Delta(\tau, \tau', \mathbf{0}) \geq 0) - [\Delta(\tau, \tau', x_{-i})|h_t]\chi(\Delta(\tau, \tau', \mathbf{0}) \leq 0)|(h_t, p_t)] \\
&= E_{t+1}[|E_t[u(\tau, x_{-i}) - u(\tau, x_{-i})|h_t]||(h_t, p_t)] \\
&= E_{t+1}[|v_t(\tau) - v_t(\tau')||(h_t, p_t)]
\end{aligned}$$

where the third line just implements the definition of the absolute value. Meanwhile, the fourth line uses the fact that when the indicators are active both functions are increasing in their arguments and the beliefs in period $t$ likelihood ratio dominate their posterior beliefs. □

Revenue Bounds, Theorem 2.. — **Proof**

In this proof, I provide bounds on equilibrium revenues. First, I provide an upper bound. Suppose that after a given period, the seller can commit to a direct, dynamic mechanism (see Myerson 1986) where buyers who have identical, initial valuations expect to have the same equilibrium payoff. Then, I find that for any dynamic mechanism, there exists a comparable mechanism that either elicits trade immediately or never trades with buyers. I show that the payoff from both mechanism are the same; meanwhile, the restriction in mechanisms ensures that it can be implemented in the current environment. The second part of the proof characterizes a lower bound. I find that there exists a collection of auctions that the seller can run in period 0 after which buyer valuations conditional on winning the good fall below the seller's valuation of $\theta_s$ and the seller might as well keep his good.

Revenue Ceiling.. — Fix some PBE, period $t$, and history $h_t$. Define a contract under full commitment as a tuple $m = (x, r, T) : \mathcal{T}^n \to \Delta^{n-1} \times \Re^n \times \mathbb{Z}_+ \cup \{\infty\}$ such that for each tuple $\tilde{\tau} = (\tau_i)$ and buyer $i$,

i. Probability that buyer $i$ wins the good is $x_i(\tilde{\tau})$

ii. Expected payment made by buyer $i$ is $r_i(\tilde{\tau})$

iii. The expected period in which agents expect the good to trade $t + T(\tilde{\tau})$.

I assume that the seller can only consider contracts in which buyers who have the same valuation for the good win the good with equal odds and make the same expected payment, i.e. for every pair of buyer $i, j$ and types $\tau_i, \tau_j$ such that $v(\tau_i, h_t) = v(\tau_j, h_t)$, it holds that

$$x_i(\tau_i, \tilde{\tau}_{-i}) = x_j(\tau_j, \tilde{\tau}_{-j}) \text{ and } r_i(\tau_i, \tilde{\tau}_{-i}) = r_j(\tau_j, \tilde{\tau}_{-j}) \tag{E26}$$

Next, I define standard functions that will allow me to state which terms of trade are feasible in a compact manner. Fix some buyer $i$ and types $\tau_i, \tau_i'$, then define the following functions

i. Discounted, payoff from buyer $i$ winning the good when he observes types $\tau_i$ but reports type $\tau_i'$:

$$q_i(\tau_i, \tau_i') \equiv E[\delta^{T(\tilde{\tau}_{-i},\tau_i')} u(\tau_i, x_{-i} \in \tilde{\tau}_{-i}) x_i(\tau_i', \tilde{\tau}_{-i}) | h_t, i \; wins] \tag{E27}$$

ii. Discounted, expected payment made by buyer $i$ when he reports type $\tau_i'$ but he is actually of type $\tau_i$:

$$p_i(\tau_i', \tau_i, B) \equiv E[\delta^{\tau(\tilde{\tau}_{-i},\tau_i')} r_i(\tilde{\tau}_{-i}, \tau_i') | h_t, i \; wins]. \tag{E28}$$

Next, the contract $m$ is feasible iff it is incentive compatible and individually rational, i.e.

$$\text{(Individual Rationality)} \qquad \forall i, \tau_i \;\; E[q_i(\tau_i, \tau_i) - p_i(\tau_i, \tau_i)] \geq 0$$

and

$$\text{(Incentive Compatability)} \qquad \forall i, \tau_i, \tau_i', \;\; q_i(\tau_i, \tau_i) - p_i(\tau_i, \tau_i) \geq q_i(\tau_i, \tau_i') - p_i(\tau_i, \tau_i').$$

Replication Argument. — Now, pick some feasible contract $m$. Define an alternative terms of trade $m'$ as follows. For every $\tilde{\tau}, i$ let

i. $\tau'(\tilde{\tau}) = 0$,

ii. $r_i'(\tilde{\tau}) = \delta^{\tau(\tilde{\tau})} r_i(\tilde{\tau})$,

iii. and $x_i'(\tilde{\tau}) = \delta^{\tau(\tilde{\tau})} x_i(\tilde{\tau})$.

Define for each buyer $i$ and types $\tau_i, \tau_i'$, the functions

$$q_i'(\tau_i, \tau_i') = E[x_i'(\tilde{\tau}_{-i}, \tau_i') u(\tau_i, \tilde{\tau}_{-i}) | h_t, i \; wins]$$

and

$$p_i'(\tau_i, \tau_i') = E[r_i'(\tilde{\tau}_{-i}, \tau_i') | h_t, i \; wins].$$

By construction, for every buyer $i$ and pair of types $\forall \tau_i, \tau_i'$, it holds that $q_i(\tau_i, \tau_i') = q_i'(\tau_i, \tau_i')$ and $p_i(\tau_i, \tau_i') = p_i'(\tau_i, \tau_i')$: thus as $m$ is feasible, then $m'$ is also feasible. Next, terms of trade $m$'s expected revenues equal to

$$r(m) = \mathbb{E}\left[\sum_{i \in I} \delta^{t(s)} r_i(s)\right] = \mathbb{E}\left[\sum_{i \in I} r_i'(s)\right] = r(m'), \tag{E29}$$

so the seller expects to gain the same returns in terms of trades $m$ and $m'$. This implies that the seller, with full commitment, might as well only consider terms of trade where he either trades in the initial period or does not trade. This establishes the proof that $r_s \leq r_s^*$ for every history almost surely.

Revenue Floor. — I now prove that the seller's revenue is greater than immediately running the revenue maximizing auction after which the seller avoids re-auctioning his good, i.e. $r_0^e < \underline{r}_0 \leq r_0$. I first establish the following technical result:

PROPOSITION 9: *There exists some value $\underline{p} \in (\theta_s, v_0(1,1))$ such that for each type $\tau$ such that $w_0(\tau)$, $E[u(\tau, x_{-i})|\forall j \neq i, w_0(\tau_j) \leq \underline{p}] \leq \theta_s$.*

Before proving this result, it is important to describe why it matters. Suppose that one dealt with a private value setting. Then, if the item fails to sell, buyers do not lower their valuation and thus there is no such $\underline{p}$ for which the seller nets higher revenues than by just running an efficient auction and this result is moot.

**Proof** This proof has several steps. First, I derive the posterior distribution on interdependent values if the seller runs an auction with a reserve price $p$, buyers decide to bid if their valuation is above $p$, but the good failed to sell. Next, I show that the initial beliefs likelihood ratio dominate these beliefs and I order posterior beliefs by $p$. I then show that valuations in period 1 are strictly increasing in the reserve price $p$. Lastly, I prove the proposition.

Bayes rule.. — Pick some pair of values $p, p' \in (v_0(0,0), v_0(1,1))$ where $p < p'$ and for each type $\tau$ $v_0(\tau) = E[u(\tau, x_{-i}|\tau_i, i \text{ wins}]$. Then for each interdependent value realization $x_i$, it holds that for each $P \in \{p, p'\}$ one can define $f(x|P) = Pr(x|m(\theta_j, x_j) \leq P)$ and it satisfies,

$$f(x|P) = \frac{Pr(v_0(\theta,x) \leq P|x)f(x)}{\int_0^1 Pr(v_0(\theta,y) \leq P|x)f(y)dy} \tag{E30}$$

Note that since $p < p'$, then for each $(x,\theta)$ such that $v_0(\theta,x) \leq p < p'$, so $Pr(v_0(\theta,x) \leq p) \leq Pr(v_0(\theta,x) \leq p')$. Furthermore, if there exists some value $\theta \in (0,1)$ such that $v_0(\theta,x) > p$, then the inequality is strict. Further notice that since $p, p' < v_1(1,1)$, then it holds that

$$\int_0^1 Pr(v_0(\theta,y) \leq p|y)f(y)dy < \int_0^1 Pr(v_0(\theta,y) \leq p'|y)f(y)dy. \tag{E31}$$

Ordering posterior beliefs by $p$.. — I claim that this fact allows me to prove that $F(\cdot|p')$, i.e. the CDF associated with $f(\cdot|p')$, first order stochastically dominates $F(\cdot|p)$.

Suppose for contradiction that there exists some value $y \in (0,1)$ such that $F(y|p) \leq F(y|p')$, then this implies that

$$\begin{aligned}
0 \leq F(y|p') - F(y|p) &= \int_0^y f(x|p') - f(x|p)dx \\
&= \int_0^y f(x)\left[\frac{Pr(v_0(\theta,x) \leq p'|x)}{\int_0^1 Pr(v_0(\theta,y) \leq p'|y)f(y)dy} - \frac{Pr(v_0(\theta,x) \leq p|x)}{\int_0^1 Pr(v_0(\theta,y) \leq p|y)f(y)dy}\right]dx \\
&\leq \int_0^y f(x)Pr(v_0(\theta,x) \leq p'|x)dx\left[\frac{1}{\int_0^1 Pr(v_0(\theta,y) \leq p'|y)f(y)dy} - \frac{1}{\int_0^1 Pr(v_0(\theta,y) \leq p|y)f(y)dy}\right] \\
&< 0\int_0^y f(x)Pr(v_0(\theta,x) \leq p'|x)dx \\
&= 0
\end{aligned} \tag{E32}$$

This is a contradiction. Note that the third line follows from the fact that $Pr(v_0(\theta,x) \leq p) \leq Pr(v_0(\theta,x) \leq p')$ and the fourth line from equation E31. This implies that $F(\cdot|p')$ first order stochastically dominates $F(\cdot|p)$.

Initial Beliefs Dominate Posteriors. — Next, I claim that $F$ likelihood ratio dominates $F(\cdot|p)$ for $P \in (v_0(0,0), v_0(1,1))$. Fix $P$, then note that for each pair $0 < x < x' < 1$, it holds that $Pr(v_0(\theta,x') \leq P|x') \leq Pr(v_0(\theta,x) \leq P|x')$ and $v_0(\cdot,\cdot)$ is a strictly increasing function. Furthermore, if $v_0(q,x) > P$, then the inequality is strict. Now, pick some pair $z, y \in [0,1]$ such that $z < y$, then the aforementioned inequality implies that

$$\frac{f(z|P)}{f(y|P)} = \left[\frac{Pr(v_0(\theta,z) \leq P|z)}{Pr(v_0(\theta,y) \leq P|y)}\right]\frac{f(z)}{f(y)} \leq \frac{f(z)}{f(y)} \tag{E33}$$

where the inequality is strict if $v_0(y,1) > P$. Therefore, $F$ likelihood ratio dominates $F(\cdot|p)$ for $P \in (v_0(0,0), v_0(1,1))$ and hence $F$ first order stochastically dominates $F(\cdot|p)$ for $P \in (v_0(0,0), v_0(1,1))$.

Beliefs are continuous in $p$.. — Pick some $p \in (v_0(0,0), v_0(1,1))$, then observe that for each value $x \in [0,1]$

$$Pr[v_0(x,\theta) \le p|x] \equiv t(p,x) = \begin{cases} 1 & if\ v(x,1) \le p \\ K[v_0^{-1}(x,p)] & if\ v(x,1) > p \end{cases} \tag{E34}$$

where $v_0^{-1}(x,\cdot)$ is well defined for each $x$ and differentiable since $u(\cdot)$ is a strictly increasing and continuously differentiable function. This implies that in all but a Lebesgue-measure zero set, the partial derivative $\partial_p t(x,p)$ is well defined for each $x$. Moreover, beliefs can be re-written as

$$f(x|p) = \frac{t(x,p)f(x)}{\int_0^1 t(y,p)f(y)dy}, \tag{E35}$$

so the partial derivative $\partial_p f(x|p)$ is well defined almost surely. Next, I define valuations given a reserve price $p \in (v_0(0,0), v_0(1,1))$ for each type $\tau$ as

$$v(\tau,p) = E[u(\tau,x_{-i})|\forall j \ne i, v_0(\tau_j) \le p] = \int_{[0,1]^{n-1}} u[\tau, x_{-i} = (x_j)_{j\ne i}] \prod_{j \ne i} f(x_j|p) d(x_j)_{j \ne i}. \tag{E36}$$

Observe that the partial derivative of $\partial_p v(\tau,p)$ is well defined and hence for each type $\tau$, $v(\tau,\cdot)$ is a continuous function since it is continuously differentiable.

Beliefs are increasing in $p$.. — I now continue with the function $v(\tau,\cdot)$ for a fixed $\tau$. I claim that for each type $\tau$, $v(\tau,\cdot)$ is strictly increasing.

Fix some $\tau \in \mathcal{T}$ and a pair of reserve prices $p, p' \in (v_0(0,0), v_0(1,1))$ such that $p < p'$. Then as $u(\cdot)$ is a strictly increasing function of $x_{-i}$ and $F(\cdot|p')$ first order stochastically dominates $F(\cdot|p)$, then

$$v(\tau,p)=\int_{[0,1]^{n-1}}u[\tau,x_{-i}=(x_j)_{j\neq i}]\prod_{j\neq i}f(x_j|p)d(x_j)_{j\neq i}$$
$$<\int_{[0,1]^{n-1}}u[\tau,x_{-i}=(x_j)_{j\neq i}]\prod_{j\neq i}f(x_j|p')d(x_j)_{j\neq i}=v(\tau,p'). \tag{E37}$$

This proves that for each $\tau$, $v(\tau,\cdot)$ is strictly increasing. Also, note that as $F$ likelihood ratio dominate $F(\cdot|p)$ for each $p\in(v_0(0,0),v_0(1,1))$, then it first order stochastically dominates it. Hence,

$$\forall\tau\in\mathcal{T},p\in(v_0(0,0),v_0(1,1)),v(\tau,p)<v_0(\tau). \tag{E38}$$

Proof Conclusion.. — I now prove that $\underline{p}>\theta_s$ exists. Fix some $\tau$ such that $v_0(\tau)\leq\theta_s$, then for each the previous result implies that $v_0(\tau,\theta_s)<v_0(\tau)\leq\theta_s$. This implies that if the seller runs an efficient auction and the item fails to sell, then valuations fall enough such that the seller keeps his good. Next, suppose that the seller picks as his reserve price $v_0(1,1)$, then with probability 1 for every type $\tau\in\mathcal{T}$ do not place a bid, i.e. $v_0(\tau)\leq v_0(1,1)$. Since almost surely no buyer bids, then beliefs do not update and $v_0(\tau)=v(\tau,v_0(1,1))$ for each type $\tau$ and it holds that there exists types such that $v(\tau,v_0(1,1))>\theta_s$; for instance, $\tau=(1,1)$. This implies that if the seller announces a price that is large enough such that all buyers wait, then beliefs do not update and the higher valuation among buyers remains higher than $\theta_s$.

Next, let us consider the types $\tau=\epsilon(1,1)$ for some $\epsilon\in[0,1]$, then the image of $v_0[\epsilon(1,1)]$ is $[v_0(0,0),v_0(1,1)]$. Hence, for each type $\tau$, there exist an $\epsilon(\tau)$ such that

$$v_0[\epsilon(\tau)(1,1)]=v_0(\tau)$$

. Moreover, by the assumption of monotone differences, then for each belief regarding $x_{-i}$ it holds that

$$E[u(\tau,x_{-i})-u(\epsilon(\tau)(1,1),x_{-i})]=0. \tag{E39}$$

This allows me to finish the proof by only considering the ray $\{\epsilon(1,1)|\epsilon\in[0,1]\}$. Define for each reserve price $p \in (v_0(0,0), v_0(1,1))$ the value $\epsilon(p)$ which solves

$$v_0[\epsilon(p)(1,1)] = p. \tag{E40}$$

Notice that as $u(\cdot)$ is strictly increasing, then $\epsilon(\cdot)$ is strictly increasing. Next, notice that $\epsilon(1) = v_0(1,1)$ and that for $v_0[\epsilon(\theta_s)] = \theta_s$ it holds that $\epsilon(\theta_s) \in (0,1)$.

I conclude the proof by defining one last function. Let let the maximum valuation when the item fails to sell when the reserve price is $p$ as $m(p) = v[\epsilon(p)(1,1), p]$ and note that $m(\cdot)$ is a strictly increasing and continuous function. As previously shown, $m(1) = v_0(1,1) > \theta_s$ and $m(\theta_s) < \theta_s$. By the intermediate value theorem, there exists value $\underline{\mathrm{p}} \in (\theta_s, 1)$ such that $m(\underline{\mathrm{p}}) = \theta_s$. This concludes the proof.

□

The next result characterizes the revenue associated with a one-shot auction and compares revenues. First, define $H_{j0}(.)$ to be the distribution of the $j^{th}$ highest valuation in period 0 and given a minimal valuation participating $p \in [\theta_s, \underline{\mathrm{p}}]$, the revenues are

$$r_0(p, H_0) = \theta_s + \int_p^{w_0(1,1)} x - \theta_s - \frac{1 - H_0(x)}{h_0(x)} dH_0^2(x) = \theta_s + \int_p^{w_0(1,1)} \phi(x) - \theta_s dH_0^1(x) \tag{E41}$$

This equation just defines the payoff to an auction in terms of the virtual value of the highest valuation, i.e. in terms of $\phi(x) = x - \frac{1-H_0(x)}{h_0(x)}$. Next, observe that in an efficient auction $p = \theta_s$, so $r_0^e = r_0(\theta_s, H_0$. Further note that $\phi(x) \leq x$ and in the case where $H_0$ is regular, the optimal auction with full commitment rest $p^*$ to solve $\phi(p^*) = \theta_s$. Hence, the efficient auction is suboptimal and increasing the reserve price in $p \in (\theta_s, p^*]$ will increase payoffs. If $p^* \leq \underline{\mathrm{p}}$, then the seller can implement his static, optimal auction and the commitment issue is moot; otherwise, it holds that $p^* > \underline{\mathrm{p}}$ and the revenue maximizing, feasible auction has a reserve price of $\underline{\mathrm{p}} > \theta_s$ and

$$r_0(p, H_0) - r_0^e = \int_{\theta_s}^{p} \phi(x) - \theta_s dH_0^1(x) > 0. \tag{E42}$$

Since for each $r_0(p, H_0) \leq \underline{\mathrm{r}}_0$, then this establishes the revenue floor assumption. □

*E2. Proof of Theorem 3.*

**Proof**

I now establish that the equilibrium is essentially unique and provide a condition for when the revenues with full commitment equal to those without it. This proof has three parts. First, I prove that the game essentially ends in finite time. Since actions are observable, this implies that the equilibrium is essentially unique. Lastly, I characterize conditions for which the seller implements his revenue maximizing terms of trade.

GAME EFFECTIVELY ENDS IN FINITE TIME.. —

LEMMA 10: *There exists a deterministic period $\hat{T} < \infty$ such that for every period $t \geq \hat{T}$, $u_t(h_t) \leq \theta_s$ for every history $h_t$ and PBE.*

**Proof**

Suppose for contradiction that there exists some PBE, period $t$, and history $h_{t+1}$ such that

$$v_t(h_{t+1}) \equiv \inf\{x \in [0,1] : H_t(x|h_t) = 1\} > \theta_s$$

and that after a large period $s \in \{1, 2, \ldots\}$ and small $\epsilon > 0$, it holds that

$$E_t[1 - H_t[v_{t+s}(h_{t+s})|h_{t+1}]^n|h_t] < \epsilon.$$

Then the seller's revenues are bounded above by the probability that the seller trades the good in the following $s$ periods and he extracts the maximum possible rents from the winner plus the static optimal period 0 rents in period $s$ times the remaining probability that a buyer remains who values the good more than the seller, i.e.

$$r_t(h_{t+1}) < v_t(h_{t+1})\epsilon + \delta^s r_0^*(1 - H_t(\theta_s|h_{t+1})^n) - \epsilon). \tag{E43}$$

Meanwhile, if the seller runs an efficient auction, i.e. he posts $p_t = \theta_s$, his revenues are greater than netting his private value $\theta_s$ times the probability that a buyer values the good more that the seller and otherwise keeping his good:

$$r_t^e \geq \theta_s[1 - H_t(\theta_s|h_{t+1})^n] + \delta\theta_s H_t[\theta_s|h_{t+1}]^n \tag{E44}$$

This implies that the difference between the expected equilibrium revenues and those attainable by immediately running the efficient auction are bounded above as

$$\begin{aligned} r_t(h_{t+1}) - r_t^e &< v_t(h_{t+1})\epsilon + \delta^s r_0^*(1 - H_t(\theta_s|h_{t+1})^n) - \epsilon) - \theta_s[1 - H_t(\theta_s|h_t)^n] - \delta\theta_s H_t[\theta_s|h_t]^n \\ &= (v_t(h_{t+1}) - \delta^s r_0^*)\epsilon - \delta\theta_s H_t[\theta_s|h_t]^n + (\delta^s r_0^* - \theta_s)[1 - H_t(\theta_s|h_t)^n]. \end{aligned} \tag{E45}$$

For sufficiently large $s$, it holds that $\delta^s r_0^* < \theta_s$ and for such values of $s$, it further holds that

$$\begin{aligned} r_t(h_t) - r_t^e &< (v_t(h_{t+1}) - \theta_s)\epsilon - \delta\theta_s H[\theta_s|h_t] \\ &\leq (v_t(h_{t+1}) - \theta_s)\epsilon - \delta\theta_s(1 - \epsilon) \\ &\leq [1 - \theta_s(1 - \delta)]\epsilon - \delta\theta_s. \end{aligned} \tag{E46}$$

For $\epsilon \in [0, \theta_s]$, it holds that $r_t(h_t) < r_t^e$. This is a contradiction, because it implies that the seller strictly prefers running an efficient auction rather than continuing with candidate equilibrium strategy. This implies that for $s$ solving $\delta^s = \theta_s$ and $\epsilon = \delta\theta_s$, it holds that by period

$$\hat{T} = \frac{1}{\delta\theta_s}\left\lceil \frac{\ln \theta_s}{\ln \delta} \right\rceil \tag{E47}$$

the item must either sells or the seller keeps his item. This concludes the proof □

Since the game essentially ends by some finite period $\hat{T}$, then all PBE can be characterized via backwards induction. As actions are further perfectly observable, then the equilibrium is essentially unique.

Before moving on, I make a quick remark. Suppose that agents interact in period $t = \Delta_t, 2\Delta_t, \ldots$ for some $\Delta_t > 0$. Then $\delta = e^{-r\Delta_t}$ for some discount factor $r > 0$ and one can construct a value $s\Delta_t$ instead of $s$ and it holds that by period $\hat{T}$, the game must end. In other words, as $\Delta_t$ goes to zero, the number of times an item is re-auctioned may diverge. But the period after which the game ends does not.

Characterizing Full Mitigation.. — I lastly prove that the condition 9 is sufficient for the seller to implement his optimal auction under full commitment. For every type $\tau_i$ excluded from the optimal static auction, it holds that $\theta_s \geq \bar{\phi}[w_0(\tau_i)]$. Therefore, if inequality 9 holds, then

$$w(\tau_i, p^*) \leq \phi(\tau_i, p^*) \leq \bar{\phi}(\tau_i, p^*) \leq \theta_s; \tag{E48}$$

where $v(\tau_i, p^*)$ is the buyer $i$'s valuation conditional on the good failing to sell when the seller posted $p_0 = p^* = p_0^*$, buyers expected to good to not re-auction the good and yet no buyer submitted a valid bid. Therefore, inequality E48 implies that the seller values the good more than his buyers and he cannot gain from re-auctioning his good as he must accept a payment below his valuation. □