When Timing Trumps Design Online Appendix

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Abstract

In this document, we provide the derivations and proofs for the paper *When Timing Trumps Design* as well as a secondary test on labor disputes that is not placed in the main text.

Appendix Overview

This appendix provides supplementary material referenced in the main text. Section 1 presents mathematical proofs for all lemmas and theoretical results. We provide detailed derivations of equilibrium dynamics and belief evolution. Section 2 presents a secondary empirical test using labor dispute data that validates our theoretical mechanisms in a different institutional context. Additional robustness checks, alternative specifications, and methodological details support the transparency and replicability of the paper's core results.

1 Proofs

1.1 Proof of Lemma 1

Let $i \in \{1, 2\}$. Suppose communications break down before time t, and the state is ℓ_i, while the state of player $-i$ is ℓ_{-i}. Since the state evolves as time-independent Markov chains, the payoff to player i can be written as a function of the joint state $\ell = (\ell_1, \ell_2)$, denoted $-B_{i\ell}$.

According to the Feynman-Kac formula, the function $B_{i\ell}$ satisfies:

$$-rB_{i\ell} = -c_{\ell_{-i}} + \nu_{\ell_i}(1 + B_{i\ell}) + \nu_{\ell_{-i}} B_{i\ell} + \sum_{\ell' \neq \ell} \lambda_{\ell\ell'}(B_{i\ell'} - B_{i\ell}) \tag{1}$$

The terms represent: $-c_{\ell_{-i}}$ is flow cost of continued dispute; $\nu_{\ell_i}(1+B_{i\ell})$: expected benefit from a decisive victory; $\nu_{\ell_{-i}} B_{i\ell}$: expected cost of decisive loss; and $\sum_{\ell' \neq \ell} \lambda_{\ell\ell'}(B_{i\ell'} - B_{i\ell})$: change in payoff from stochastic state transitions. Rearranging and isolating $B_{i\ell}$, we obtain:

$$B_{i\ell} = \frac{c_{\ell_{-i}} - \nu_{\ell_i} + \sum_{\ell' \neq \ell} \lambda_{\ell\ell'} B_{i\ell'}}{r + \nu_{\ell_i} + \nu_{\ell_{-i}} + \sum_{\ell' \neq \ell} \lambda_{\ell\ell'}} \tag{2}$$

Define the vector:

$$c := \left\{ \frac{c_{\ell_{-i}} - \nu_{\ell_i}}{r + \nu_{\ell_i} + \nu_{\ell_{-i}} + \sum_{\ell' \neq \ell} \lambda_{\ell\ell'}} \right\}_{\ell=1}^{n} \tag{3}$$

and the matrix:

$$\Lambda := \left[\frac{\lambda_{\ell\ell'} \cdot 1_{\ell \neq \ell'}}{r + \nu_{\ell_i} + \nu_{\ell_{-i}} + \sum_{\ell' \neq \ell} \lambda_{\ell\ell'}} \right]_{\ell,\ell'=1}^{n} \tag{4}$$

Here, Λ is a strictly sub-stochastic matrix, meaning all entries are non-negative and each row sums to less than one. Then the system becomes:

$$\mathbf{B}_i = \Lambda \mathbf{B}_i + c \quad \Rightarrow \quad \mathbf{B}_i = (I - \Lambda)^{-1} c \tag{5}$$

By the Perron-Frobenius theorem, since Λ is substochastic and $c > 0$, it follows that $\mathbf{B}_i$ exists and all entries are strictly positive. This concludes the proof.

1.2 Proof of Lemma 2

This section provides the proof for lemma 2. Intuitively, the theorem states that in every equilibrium, at most one player concedes at unstable, strategic times; otherwise, the players are indifferent between conceding and making demands.

Preliminaries In every equilibrium, a strategic player $i \in \{1, 2\}$ follows a stopping time τ_i denoting when i concedes, conditional on no prior agreement, communication breakdown, or decisive resolution for either side. Since τ_i must be adapted to the natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ generated by $\{\nu_{it}, c_{it}\}_{i=1}^{2}$, it can be described by a tuple $\{H_{it}\}$, where at each time t and history h_t, $H_{it}(h_t)$ denotes the probability that i has conceded by time t. Define i's stable time t equilibrium payoff as W_{it}. Since an equilibrium is assumed to exists, define the set of optimal, adapted stopping times: $S_i \equiv \{\tau \mid U_{it} = \sup_{\tau} E_t[U_{i\tau}]\}$. Define T_i as the first time at which i concedes: $T_i \equiv \inf\{\tau : H_{i\tau} = 1\}$.

Proof of (I): Players concede for certain at the same time This part of the lemma claims that $T_1 = T_2$ i.e., players end up conceding by the same stopping time. Intuitively, if player i is certain that j is obstinate, then she faces the decision of when she wishes to stop fighting and concede. Waiting can allow her to attain a decisive, exogenous victory, but it is costly to continue the dispute and i faces the risk that j breaks the lines of communication. This latter outcome is known to be strictly worse than conceding immediately. Since disputes are costly, then these costs of maintaining the dispute are larger than the potential benefit of attaining a victory.

Suppose, for contradiction, that some player i is certain that j is obstinate at some time t. Player i nets a payoff of 0 if she concedes immediately i.e. the payoff from conceding. Otherwise, i could follow a strategy described by some admissible stopping time $\tau_i \gg t$ and its payoff is

$$E_t(U_{i\tau_i}) = E_t\Big[e^{-r(\tau^*-t)}\chi(i \text{ wins}) - (C_{it} - C_{i0})\Big] \leq E_t\Big[e^{-r(\tau^*-t)}\chi(i \text{ wins}) - \int_t^{\tau^*} e^{-r(s-t)} c_{is}\psi_{is} \mathrm{ds}\Big]$$
$$= E_t\Big[\chi(i \text{ wins})\Big(e^{-r(\tau^*-t)} - \int_t^{\tau^*} c_{is}\psi_{is}e^{-r(s-t)}\mathrm{ds}\Big) - \chi(j \text{ wins or } i \text{ gives up}) \int_t^{\tau^*} c_{is}\psi_{is}e^{-r(s-t)}\mathrm{ds}\Big]$$
$$< E_t\Big[\chi(i \text{ wins})\Big(e^{-r(\tau^*-t)} - \int_t^{\tau^*} c_{is}\psi_{is}e^{-r(s-t)}\mathrm{ds}\Big)\Big]$$

where τ^* is the earliest time where the dispute ends either by an exogenous, decisive resolution or with i conceding. The right-hand side of the first inequality disregards the flow costs of negotiation and it is (thus) an upper bound on i's attainable payoff. The second inequality breaks the expectation into two parts: the case where i eventually wins and when she does not. Meanwhile, the third inequality exploits the fact that the payoff when i does not win is negative since the costs of disputing is strictly positive. Lastly, note that since the arrival of a decisive victory is distributed following a time and state-dependent Poisson rate, then the right-most side of the inequality can be rewritten as

$$E_t(U_{i\tau_i}) < E_t\big[\chi(i \text{ wins})\big(e^{-r(\tau^*-t)} - \int_t^{\tau^*} c_{is}\psi_{is}e^{-r(s-t)}\mathrm{ds}\big)\big]$$
$$= E_t\big[\chi(i \text{ wins}) \int_t^{\tau^*} (\nu_{is} - c_{is})\psi_{is}e^{-r(s-t)}\mathrm{ds}\big] \leq 0$$

Hence, the payoff from not giving up immediately and following any strategy other than conceding immediately yields player i strictly less payoff than conceding at time t. Hence, i concedes as soon as she is certain that j is obstinate, so $T_1 = T_2$. This establishes point (I).

Implication 1 *This lemma implies a standard boundary condition that equilibrium beliefs must satisfy: after every history, beliefs must converge to 1 at the same time. This boundary condition, alongside with the fact that not conceding only serves to increase beliefs that a* *player is obstinate, pins down the probabilities of a concession at time* 0 *and at unstable, strategic times* $t > 0$*.*

Proof of (II): At time 0 or at unstable time $t > 0$, at most one player concedes with a positive probability The following result establishes that during strategic, unstable times, concession behavior must be continuous. Suppose that at some strategic, stable time t or at time 0, i expects that j concedes (unconditional of j being strategic) with probability $p > 0$. If i concedes, she nets a payoff of 0; otherwise, she can decide to not concede at time t. Her payoff from not conceding is weakly greater than deciding to concede at time t^+ (i.e., right after time t), which nets her a payoff of p. Hence, i does not concede at time t, which implies that it cannot be the case that at time 0 or at some unstable time $t > 0$, both combatants concede. This establishes point (II).

Next, this observation implies that if i expects j to concede at a strategic, stable time t, then $W_{it} > 0$. Since the state, the rate at which communication breaks down, *and* the rate at which an exogenous resolution arrives favoring either side are distributed exponentially, then W_{it} is a continuous function of time t i.e., $W_i(\cdot)$ is continuous on every interval $[s, t]$, for $0 \leq s < t$, if for each $\tau \in [s, t]$ is a stable time. By continuity of W_{it}, there exists some small $\Delta_t > 0$ such that $\inf_{s \in [t-\Delta_t, t]} W_{is} > 0$, so strategic i strictly prefers not to concede in the time interval $[t - \Delta_t, t]$. This last observation is useful below and is summarized in a corollary below.

Corollary 1 *Suppose that i expects that j concedes with a strictly positive probability at stable time $t > 0$, then there exists a small $\Delta_t > 0$ such that i does not concede in stable times $s \in [t - \Delta_t, t]$.*

Implication 2 *This result, coupled with the fact that not conceding can only increase beliefs that a player is obstinate since obstinate players never concede, implies that at least one player does not concede at an unstable time. In addition, with a positive probability, the opponent's belief that said player is obstinate converges to 1 before the state shifts. Consequently, the probability that a player concedes is simple set so that their opponent's beliefs converge to 1 at the same time as their own.*

Proof of (III): At strategic, stable times $t > 0$, concessions must arrive gradually This is the first result leading to point (III) in the lemma. Intuitively, the result clarifies that concession behavior must be gradual at strategic, stable times $t > 0$. First, fix some pair of strategic stable times t, s such that $t < s$ and suppose that i expects that j does not concede in the interval $[t, s]$. The claim is that i also chooses to not concede in the given interval. Her time t payoff from conceding at time t is 0; meanwhile, her payoff from conceding at time $t' \in (t, s]$ is

$$E_t\big[\chi(i \text{ wins at time } w \in (t,t'])(e^{-r(w-t)} - (C_{iw} - C_{it})) + \chi(j \text{ wins at time } w \in (t,t'])(C_{iw} - C_{it}) \\ - \chi(\text{communication breaks down}) B_{i\ell_t} - (C_{it'} - C_{it})\chi(i \text{ concedes before subsequent event})\big] \\ < E_t[\chi(i \text{ wins at time } w \in (t,t'])(e^{-r(w-t)} - (C_{it'} - C_{it}))] < 0 \quad (1)$$

where ℓ_t is the state at stable time t. The inequality above points out that $B_{i\ell_t} > 0$, $(C_{iw} - C_{it}) > 0$, and $(C_{it'} - C_{it}) > 0$, so i's payoff from conceding at time t' is less than the payoff from her decisively winning before time t'. Lastly, the second inequality follows from the same logic described in the subsection establishing point (I). Since i at time t, nets a payoff of 0 from conceding right away, which is strictly higher than the expected payoff from conceding at any time $t' \in (t, s]$, then (in equilibrium) she never concedes at times $t' \in (t, s]$. This corollary is summarized below.

Corollary 2 *If i does not concede at stable times $\tau \in (t, s]$ for $0 \leq t < s$, then neither does j.*

No discontinuous concessions at strategic, stable times Next, this observation implies that there does *not* exists a stable, strategic time $t > 0$ such that some strategic player i concedes with a strictly positive probability. Suppose, for contradiction, that i expects j to concede at some stable, strategic time $t > 0$. Corollary 1 implies that i strictly prefers to not concede with a strictly positive probability at stable times $\tau \in [t - \Delta_t, t]$. In response, corollary 2 implies that j would strictly prefer to concede at time s rather than t, which contradicts i's assumption that j would concede at time t with a strictly positive probability. This establishes point (III).

Proof of (IV): Sub-game equilibrium payoffs equal conceding immediately with a positive probability This final part of the proof establishes that both players are indifferent between conceding and making demands at each strategic, stable time $t > 0$. Fix a strategic time $t > 0$ and a history h_t. We claim that for any $t \leq s < \tau$, it must hold that either $H_{is}(h_t) < H_{i\tau}(h_t)$ or $H_{is}(h_t) = H_{i\tau}(h_t) = 1$ for each $i \in \{1, 2\}$. Intuitively, this follows from the fact that players randomize between conceding and making demands at each stable time, which can only occur if they are indifferent between these actions.

Proof by Contradiction Suppose, for contradiction, that there exists some strategic time $t > 0$ and history h_t such that $H_{is}(h_t) = H_{i\tau}(h_t) < 1$ for some player i and some $0 < t < s < \tau$.

Define $T \equiv \sup\{\tau > s \mid H_{is}(h_t) = H_{i\tau}(h_t), \tau \text{ is a stable time}\}$. By assumption, i does not concede at any stable time $\tau \in [s, T]$.

By Corollary 2, this implies that player j is also unwilling to concede at any stable time $\tau \in [s, T]$. Consequently, if i expects that j does not concede in this interval, then i strictly prefers to concede at some time within $[T - \Delta_t, T]$ for some sufficiently small $\Delta_t > 0$, meaning that $H_{iT}(h_t) > H_{is}(h_t)$.

However, by Lemma 1, point (III), we know that i does not concede at time T with strictly positive probability. Therefore, it must hold that $H_{iT}(h_t) = \lim_{\epsilon \searrow 0} H_{iT-\epsilon}(h_t)$. This contradicts the definition of T as the supremum of stable times where $H_{is}(h_t) = H_{i\tau}(h_t)$. Thus, the original assumption must be false, completing the proof of point (IV). This concludes the proof.

Implication 3 *The key implication of this result is that one can use this expression for payoffs and the fact that payoffs are constant to pin down concession dynamics. The following section uses this observation to characterize concession dynamics.*

1.3 Equilibrium Derivation

In this part of the appendix, we derive the equilibrium payoffs and the concession dynamics. Notice that we characterize belief dynamics in the paper's in-text appendix.

Equilibrium Payoffs and Concession Rates Let $W_{i\ell t}$ denote the expected discounted payoff to player i at a strategic, stable time $t > 0$, conditional on state ℓ. Because the underlying process is a continuous-time Markov chain and type behavior evolves independently, $W_{i\ell t}$ satisfies the following Feynman–Kac equation:

$$\begin{aligned} rW_{i\ell t} &= -(\bar{c} + \psi_t c_{j\ell}) + \mu_{jt}\phi(-B_{i\ell} - W_{i\ell t}) + C_{j\ell t}(1 - W_{i\ell t}) \\ &+\psi_t \nu_{i\ell}(1 - W_{i\ell t}) + \psi_t \nu_{j\ell}(-W_{i\ell t}) + \sum_{\ell' \neq \ell} \lambda_{t\ell\ell'} \left[(1 - q_{j\ell\ell' t})(1 - W_{i\ell t}) + q_{j\ell\ell' t}(W_{i\ell' t^+} - W_{i\ell t})\right] + \dot{W}_{i\ell t}. \end{aligned} \tag{2}$$

At stable, strategic times, Lemma 2 implies players are indifferent between conceding and delaying. Hence, we set $W_{i\ell t} = \dot{W}_{i\ell t} = 0$. Let $K_{i\ell t}$ denote the total concession rate: $K_{i\ell t} \equiv C_{i\ell t} + \sum_{\ell' \neq \ell}(1 - q_{i\ell\ell' t})\lambda_{t\ell\ell'}$. Substituting into Equation (2), we obtain the equilibrium expression:

$$K_{i\ell t} = \bar{c} + \mu_{jt}\phi B_{i\ell} + \psi_t(c_{i\ell} - \nu_{j\ell}),$$

as reported in the main text.

Concession Probabilities and Belief Convergence Let t denote a moment of state transition. By Lemma 2, at most one player concedes discontinuously at such a point. Without loss of generality, suppose player i is the potential conceding party. Let T_{it} denote the time it would take for player j's belief about i's obstinacy to converge to certainty if no further transitions occur. This convergence time satisfies:

$$0 \quad = \quad \ln \mu_{it^-} \;+\; \int_t^{T_{it}} \frac{\mathrm{d}\ln \mu_{is}}{\mathrm{ds}}\,\mathrm{d}s \quad = \quad \ln \mu_{it^-} \;+\; \int_0^{T_{it}} \left(C_{i\ell t+s} - \phi(1-\mu_{it+s})\right)\,\mathrm{d}s, \quad (3)$$

where μ_{it^-} is player j's belief just prior to t. Let $T_t = \min_i T_{it}$ denote the equilibrium convergence horizon. Next, suppose player i does not concede at time t, updating player j's belief via Bayes' Rule: $\ln \mu_{it} = \ln \mu_{it^-} - \ln q_{it}$. To ensure that belief convergence still occurs by $t + T_{it}$, the updated belief must satisfy:

$$0 = \ln \mu_{it^-} - \ln q_{it} + \int_0^{T_{it}} \left(C_{i\ell t+s} - \phi(1-\mu_{it+s})\right)\,\mathrm{d}s. \quad (4)$$

Combining Equations (3) and (4) yields:

$$q_{it} = \left(\frac{\mu_{it^-}}{\mu_{jt^-}}\right) \exp\left\{\int_0^{T_{it}} \left[C_{i\ell t+s} - C_{j\ell t+s} + \phi(\mu_{it+s} - \mu_{jt+s})\right]\,\mathrm{d}s\right\}. \quad (5)$$

This expression defines the equilibrium concession probability q_{it} in terms of reputation dynamics and belief convergence paths. Intuitively, the player whose reputation converges more slowly must concede more often to avoid being mistaken for an obstinate type. The recursive nature of this dynamic underpins the broader logic of delay: reputations and transition expectations jointly shape who concedes, when, and why.

Backward Induction To complete the equilibrium characterization, we derive closed-form expressions for the full set of strategic quantities: $\left\{\{C_{i\ell t},\, q_{i\ell\ell' t},\, \mu_{it}\}_{i=1}^{2},\, T_t\right\}$. Although the preceding results establish functional relationships among these objects, full determination requires recursive settlement across the state space. We proceed via backward induction on the leverage states.

Step 1: Terminal State $\ell = n$. Suppose the process reaches the terminal state $\ell = n$ at some time t, with $\max_i \mu_{it^-} < 1$. Since no further state transitions are possible, the total concession rate simplifies to: $C_{int+s} = \bar{c} + \mu_{it+s}\phi B_{jn} + \psi_{t+s}(c_{in} - \nu_{jn})$. This closed-form allows direct computation of belief dynamics via Equations

$$\frac{\mathrm{d}\ln\mu_{it}}{\mathrm{dt}} = C_{it} - \phi(1-\mu_{it}) \tag{6}$$

, (4), and (5). Crucially, because no future states exists beyond $\ell = n$, all post-jump concessions are fully determined by the destination state $\ell' = n$, not the origin $\ell < n$. Thus, for any such jump, we write $q_{i\ell nt} = q_{int}(\mu_{it^-}, \mu_{jt^-})$, abstracting from the prior state.

Step 2: Inductive Step. Suppose equilibrium quantities—$C_{i\ell t}$, $q_{i\ell\ell' t}$, T_t, and $\dot{\mu}_{it}/\mu_{it}$—have been derived for all states $\ell' = m, \ldots, n$. Then, for state $\ell = m-1$, the total concession rate satisfies:

$$C_{i\,m-1\,t} = \bar{c} + \mu_{it}\phi B_{j\,m-1} + \psi_t(c_{i\,m-1} - \nu_{j\,m-1}) - \sum_{\ell'=m}^{n}(1 - q_{i\,\ell'\,t})\lambda_{t\,m-1\,\ell'}. \tag{7}$$

Because the concession probabilities $q_{i\,\ell'\,t}$ for $\ell' \geq m$ are known by induction, Equation (7) uniquely determines $C_{i\,m-1\,t}$. Applying Equation (8) then yields $\dot{\mu}_{it}/\mu_{it}$. These quantities together permit backward computation of $q_{i\,m-1\,t}$ and T_t for prior states $\ell < m-1$, via Equations (4) and (5).

As in the terminal case, concession probabilities depend only on the jump destination $\ell' = m-1$ and pre-jump beliefs. The origin state ℓ is irrelevant to equilibrium behavior at time t. Accordingly, we write $q_{i\ell\,m-1\,t} = q_{i\,m-1\,t}(\mu_{it^-}, \mu_{jt^-})$.

Conclusion. Iterating this procedure from $\ell = n$ down to $\ell = 1$ fully determines all strategic quantities in equilibrium.

1.4 Learning and Diminishing Clustering

We now present a more rigorous derivation of the belief dynamics than presented in the main text. We delegate these calculation here for two reasons. First, the logic is somewhat involved and technical and it is thus likely to complicate the paper's presentation. But, two, the logic presented herein is central to the dynamics pinning down our formal results.

Evolution of Beliefs Before deriving an expression for $q_{i\ell\ell' t}$, it is necessary to derive an expression for beliefs. Let μ_{it} denote player j's belief that i is obstinate at time t. The evolution of these beliefs differ between stable and unstable strategic times t. At unstable times, beliefs update discontinuously. If player i fails to concede at unstable, strategic time

t when the state shifts from ℓ to ℓ' (for $\ell' > \ell$), then $j$updates her beliefs via Bayes rule as

$$\mu_{it} = \frac{\mu_{it^-}}{q_{i\ell\ell' t}}$$

since $q_{i\ell\ell' t}$ is the probability that i does not concede at time t.

During stable phases, beliefs evolve gradually. Player j's belief that i is obstinate increases as j fails to observe a concession, but this effect is attenuated by failing to observe communication breaking down i.e., Bayes rule implies the following belief updating rule:

$$\frac{d}{dt} \ln \mu_{it} = C_{it} - \phi(1 - \mu_{it}). \tag{8}$$

This expression clarifies that players only build a reputation for being non-strategic when the dispute is sufficiently costly. Employing equations 8, the expression for the total concession rate provided in the main text, and the expression for belief dynamics, we assume the following condition ensuring that players can build a reputation over time:

Assumption 1

$$\bar{c} \geq \phi + \max_{\ell \in \{1,\ldots,L\}} \sum_{\ell' > \ell} \lambda_{\ell' \ell} \tag{9}$$

Boundary Condition Next, beliefs must converge to 1 simultaneously. Suppose, for contradiction, i becomes certain that j is obstinate. If i does not concede immediately, then she never benefits from conceding. However, her payoff from not conceding is equivalent to her payoff from not being able to concede since communication broke down i.e. $-B_{i\ell} < 0$. Since her payoff from conceding is 0, we have a contradiction.

This convergence condition pins down the probability of immediate concession at time 0 and at unstable times. Since beliefs evolve at different rates across players, the player whose belief converges faster will be the one to concede. Anticipating this, the other player delays. Thus, at most one player concedes discontinuously at time 0 or any strategic shift. If both were willing to concede, each would prefer to wait for the other—a contradiction.

Asymmetry matters The asymmetry is central. Because only one player concedes at a transition, only that player reduces her gradual concession rate beforehand. The other anticipates no discrete concession and thus maintains pressure. This creates intertemporal distortion—particularly when $\psi_t = 0$, which selectively mutes early pressure and shifts the concession effort forward. Even when the policy is formally neutral, the timing asymmetry creates strategic inequality.

Discrete Concession Behavior We lastly characterize the term $q_{i\ell\ell' t}$. If the state shifts from ℓ to ℓ' at a strategic time $t > 0$, then define the smallest time when beliefs simultaneously converge to 1 if the state does not shift further as $T_t > 0$. If player i is the one conceding, then it holds that

$$0 = \ln \mu_{it^-} - \ln q_{i\ell\ell' t} + \int_t^{t+T_t} \frac{d}{ds} \ln \mu_{is} \text{ds} = \ln \mu_{jt} + \int_t^{t+T_t} \frac{d}{ds} \ln \mu_{js} \text{ds} \tag{10}$$

To fully derive an expression for $q_{i\ell\ell' t}$, we conclude as follows. When $\ell = L$, it is guaranteed that $C_{iLt} = K_{iLt}$ and thus a closed-form expression can be provided for the evolution of beliefs stated in equation 8. In turn, the evolution of beliefs pins down the earliest time when both player's beliefs converge to 1 i.e., T_t. Similarly, an expression for T_t and the gradual evolution of beliefs pins down $q_{i\ell Lt}$ for each ℓ and prior beliefs (μ_{1t}, μ_{2t}). To conclude, we proceed fully deriving expressions for $C_{i\ell t}, q_{i\ell\ell' t}$, and T_t via backwards induction, but derivations appear in the previous sections.

1.5 Proof of Lemma 4: The De-escalation Paradox

In this section, we prove that increasing the amount of time in which $\psi_t = 1$, reduces the expected duration conditional and the time spent fighting. The first part of the proof derived an expression for the expected, duration of a dispute. Meanwhile, the second part of the proof derives an expression for the initial concession probability.

Outline We proceed in three steps. First, we derive the expected dispute duration $d_{\ell t}$. Second, we simplify this expression under de-escalation. Third, we analyze the limit as $\mu \to 0$ to obtain the comparative statics in Lemma 4.

Expected Duration The proof for the derivation of the expected duration of the dispute has two steps. First, we derive an expression for the duration of a dispute when the lines of communication previously broke down. This demonstration can be derived via backwards induction by exploiting the fact that once communication breaks down, all objects are time homogeneous. Second, we use the expressions derived to provide an expression for the duration of disputes overall via backwards induction.

Duration We now move to define and then characterize the expected duration of high intensity disputes. Let $\tau \geq 0$ be the time when the dispute ends, then let $d_{\ell t} \equiv E_t[\tau]$ be the expected duration of the dispute conditional on the dispute not ending by stable, strategic time $t \geq 0$ and the state being ℓ. Using the Feynman-Kac formula implies that

$$0d_{\ell t} = \sum_i (C_{i\ell t} + \psi_t \nu_{i\ell})[t - d_{\ell t}] + \sum_{\ell' > \ell} \lambda_{\ell \ell' t}[(1 - q_{i\ell' t}) + q_{i\ell' t} d_{\ell' t} - d_{\ell t}] + \mu_{i\ell t}\phi[A_\ell - d_{\ell t}] + \dot{d}_{\ell t} \quad (11)$$

Until the dispute ends, the dispute's duration increases by a constant flow of unity (i.e., $E_t[\tau] = E_t[\int_0^\tau 1\text{dt}]$). The second part of the right-hand side of the equation clarifies that the dispute can end immediately due to one party conceding or said party attaining a decisive victory. However, the third part clarifies that the state can shift before the dispute ends gradually. When the state shifts, one of the parties may concede with a positive probability or the dispute continues in a different state. Next, the equation clarifies that the lines of communication can be broken, which leads to a payoff of A_ℓ.

If one regroups terms, it holds that

$$-\left[\sum_i \mu_{i\ell t}\phi A_\ell + (K_{i\ell t} + \nu_{i\ell}\psi_t)t + \sum_{\ell' > \ell} \lambda_{\ell\ell' t} q_{i\ell' t} d_{\ell' t}\right] = -\left[\sum_i (C_{i\ell t} + \psi_t \nu_{i\ell} + \mu_{i\ell t}\phi) + \sum_{\ell' > \ell} \lambda_{\ell \ell' t}\right] d_{\ell t} + \dot{d}_{\ell t} \quad (12)$$

Adding 2ϕ and subtracting ϕ for each i in the second bracket, then implies that

$$-\left[\sum_i \mu_{i\ell t}\phi A_\ell + t(K_{i\ell t} + \nu_{i\ell}\psi_t) + \sum_{\ell' > \ell} \lambda_{\ell\ell' t} q_{i\ell' t} d_{\ell' t}\right] = -\left[2\phi + \sum_i \frac{\dot{\mu}_{i\ell t}}{\mu_{i\ell t}} + \psi_t \nu_{i\ell} + \sum_{\ell' > \ell} \lambda_{\ell \ell' t}\right] d_{\ell t} + \dot{d}_{\ell t} \quad (13)$$

Using standard arguments, this equation can be further rewritten as

$$\begin{aligned} &\frac{\text{d}}{\text{dt}}\left[\frac{e^{-2\phi t - \sum_i \nu_{i\ell}\int_0^t \psi_s \text{ds} + \int_0^t \sum_{\ell' > \ell} \lambda_{\ell\ell' s}\text{ds}} d_{i\ell t}}{\prod_i \mu_{i\ell t}}\right] \\ &\quad = -\frac{e^{-2\phi t - \sum_i \nu_{i\ell}\int_0^t \psi_s \text{ds} + \int_0^t \sum_{\ell' > \ell} \lambda_{\ell\ell' s}\text{ds}}}{\prod_i \mu_{i\ell t}}\left[\sum_i \mu_{i\ell t}\phi A_\ell + t(K_{i\ell t} + \nu_{i\ell}\psi_t) + \sum_{\ell' > \ell} \lambda_{\ell\ell' t} q_{i\ell' t} d_{\ell' t}\right] \quad (14) \end{aligned}$$

Integrating both sides of the equation implies that there exists a constant Ψ_ℓ such that

$$\frac{e^{-2\phi t-\sum_i \nu_{i\ell}\int_0^t \psi_s \mathrm{ds}+\int_0^t \sum_{\ell'>\ell}\lambda_{\ell\ell' s}\mathrm{ds}} d_{i\ell t}}{\prod_i \mu_{i\ell t}} =$$

$$\Psi_\ell - \int_0^t \frac{e^{-2\phi s-\sum_i \nu_{i\ell}\int_0^s \psi_w \mathrm{dw}+\int_0^s \sum_{\ell'>\ell}\lambda_{\ell\ell' w}\mathrm{dw}}}{\prod_i \mu_{i\ell s}}\left[\sum_i \mu_{i\ell s}\phi A_\ell + t(K_{i\ell s}+\nu_{i\ell}\psi_s) + \sum_{\ell'>\ell}\lambda_{\ell\ell' s} q_{i\ell' s} d_{\ell' s}\right]\mathrm{ds} \tag{15}$$

Note that $\tau \geq 0$, so for each state ℓ and time $t \geq 0, d_{\ell t} \in [0,1]$. In turn, this observation implies that as $t \to \infty$, the left-hand side of the equation above converges to 0 and

$$\Psi_\ell = \int_0^\infty \frac{e^{-2\phi s-\sum_i \nu_{i\ell}\int_0^s \psi_w \mathrm{dw}+\int_0^s \sum_{\ell'>\ell}\lambda_{\ell\ell' w}\mathrm{dw}}}{\prod_i \mu_{i\ell s}}\left[\sum_i \mu_{i\ell s}\phi A_\ell + t(K_{i\ell s}+\nu_{i\ell}\psi_s) + \sum_{\ell'>\ell}\lambda_{\ell\ell' s} q_{i\ell' s} d_{\ell' s}\right]\mathrm{ds} \tag{16}$$

Hence, the value functions satisfy that

$$d_{\ell t} = \int_t^\infty \prod_i \frac{\mu_{i\ell t}}{\mu_{i\ell s}} e^{-2\phi(s-t)-\sum_i \nu_{i\ell}\int_t^s \psi_w \mathrm{dw}+\int_t^s \sum_{\ell'>\ell}\lambda_{\ell\ell' w}\mathrm{dw}} \times \left[\sum_i \mu_{i\ell s}\phi A_\ell + t(K_{i\ell s}+\nu_{i\ell}\psi_s) + \sum_{\ell'>\ell}\lambda_{\ell\ell' s} q_{i\ell' s} d_{\ell' s}\right]\mathrm{ds} \tag{17}$$

If one then sets $\lambda_{\ell\ell' t} = 0$ for each pair of states ℓ and ℓ' and time t, the equation above becomes:

$$d_{\ell t} = \int_t^\infty \prod_i \frac{\mu_{i\ell t}}{\mu_{i\ell s}} e^{-2\phi(s-t)-\sum_i \nu_{i\ell}\int_t^s \psi_w \mathrm{dw}}\left[\sum_i \mu_{i\ell s}\phi A_\ell + t(K_{i\ell s}+\nu_{i\ell}\psi_s)\right]\mathrm{ds} \tag{18}$$

This expression shows how longer periods of de-escalation (i.e., low ψ_t) increase expected duration, as fewer incentives to settle accumulate. If one then set $\phi = 0$ and sets $t = 0$ and $\ell = 1$, then define $d^* \equiv d_{10}$ and the equation above further simplifies to

$$d^* = \int_0^\infty \prod_i \frac{\mu_{i10+}}{\mu_{i1t}} e^{-\sum_i \nu_{i1}\int_0^t \psi_s \mathrm{ds}} t\left[\sum_i K_{i1t} + \nu_{i1}\psi_t\right]\mathrm{dt} \tag{19}$$

Note that as $(\lambda_{\ell\ell' t}) = 0$ and $\phi = 0$ then $K_{i\ell t} = C_{i\ell t} = \frac{\mathrm{d}}{\mathrm{dt}}\ln \mu_{i\ell t}$ due to the expressions given in the main text. Using these observations in the equation above implies that

$$d^* = -\prod_i \mu_{i10+}\int_0^\infty t\frac{\mathrm{d}}{\mathrm{dt}}\left[e^{-\sum_i \ln \mu_{i1t}-\sum_i \nu_{i1}\int_0^t \psi_s \mathrm{ds}}\right]\mathrm{dt} \tag{20}$$

The integral can be further simplified using integration by parts yielding that

$$d^* = \prod_i \mu_{i10^+}\left[te^{-\int_0^t \psi_s \mathrm{ds} \sum_i \nu_{i1}} \prod_i \frac{1}{\mu_{i1t}} |_0^\infty - \int_0^\infty e^{-\sum_i \nu_{i1}\int_0^t \psi_s \mathrm{ds}} \prod_i \frac{1}{\mu_{i1t}} \mathrm{dt}\right] \tag{21}$$

Next, observe that since $\phi = 0$ and $(\lambda_{\ell\ell' t}) = 0$, then for each player i

$$\ln \mu_{i1t} = \ln \mu_{i10^+} + \int_0^t \frac{\dot{\mu}_{i1s}}{\mu_{i1s}} \mathrm{ds} = \ln \mu_{i10^+} + \int_0^t K_{i1s} \mathrm{ds} \tag{22}$$

or that for each player i,

$$\frac{1}{\mu_{i1t}} = \frac{e^{-\int_0^t K_{i1s}\mathrm{ds}}}{\mu_{i10^+}} \tag{23}$$

Plugging this observation into equation 21 implies that

$$d^* = te^{-\int_0^t \sum_i \nu_{i1}\psi_s + K_{i1s}\mathrm{ds}} |_0^\infty - \int_0^\infty e^{-\int_0^t \sum_i \nu_{i1}\psi_s + K_{i1s}\mathrm{ds}} \mathrm{dt} \tag{24}$$

Next, observe that when $\phi = \bar{c} = 0$, then $K_{i1s} = (c_{j1} - \nu_{i1})\psi_s$ for each s and i, which further implies that

$$d^* = te^{-\int_0^t \sum_i c_{i1}\psi_s \mathrm{ds}} |_0^\infty - \int_0^\infty e^{-\int_0^t \sum_i c_{i1}\psi_s \mathrm{ds}} \mathrm{dt} \tag{25}$$

This expression can be further rewritten as

$$d^* = -\int_0^\infty e^{-\int_0^t \psi_s \mathrm{ds} \sum_i c_{i1}} \mathrm{dt} + \lim_{t\to\infty} te^{-\int_0^t \psi_s \mathrm{ds} \sum_i c_{i1}} \tag{26}$$

This concludes the expression for the expected duration of dispute conditional on the dispute not ending from the outset.

Probability of dispute The second result calculates the probability that one of the players concedes from the outset. Let the probability that i concedes from the outset be $q_{i0} = \min\{1, q_i\}$ where the main text shows that

$$q_i \mu = \mu e^{\int_0^{T_0} (C_{i1s} - C_{j1s} + \phi(\mu_{i1s} - \mu_{j1s}))\mathrm{ds}} \tag{27}$$

Next, the minimum amount of time until i's beliefs converge to 1 starting from μ is T_{i0} where

$$-\ln \mu = \int_0^{T_{i0}} (C_{is} - \phi(1 - \mu_{is}))\, \mathrm{ds}. \tag{28}$$

As $\phi = 0$, it holds that

$$q_i = e^{\int_0^{T_0} C_{i1s} - C_{j1s} \mathrm{ds}} \tag{29}$$

and since it further holds that $(\lambda_{\ell\ell' t}) = 0$, then $C_{i1s} = K_{i1s} = \bar{c} + (c_{j1} - \nu_{i1})\psi_s$ implying that

$$q_i = e^{-[(\nu_{i1}-\nu_{j1})+(c_{i1}-c_{j1})]\int_0^{T_0} \psi_s \mathrm{ds}} \tag{30}$$

and that T_{i0} satisfies that

$$-\ln \mu = \bar{c}T_{i0} + (c_{j1} - \nu_{i1}) \int_0^{T_{i0}} \psi_s \mathrm{ds}. \tag{31}$$

and $T_0 = \min_i T_{i0}$. Next, define the probability that either side concedes as $q_0(\mu, \{\psi_t\}) = \min_i q_{i0}$ and let the $T_0(\mu) = \min_i T_{i0}(\mu)$ implicitly denote the time T_{i0} solving equation 31 , so

$$q^*(\mu, \{\psi_t\}) = e^{-[(\nu_{11}-\nu_{21})+(c_{11}-c_{21})]\int_0^{T_0(\mu)} \psi_s \mathrm{ds}} \tag{32}$$

Observe that as $\mu \to 0, T_{i0}(\mu) \to \infty$ for each player i, so if $q^* = \lim_{\mu \searrow 0} q_0(\mu, \{\psi_t\})$, then

$$q^* = e^{-[(\nu_{11}-\nu_{21})+(c_{11}-c_{21})]\int_0^{\infty} \psi_s \mathrm{ds}} \tag{33}$$

This concludes the proof.

1.6 Proof of Lemma 5

We now derive for each $i \in \{1, 2\}$'s bargaining power.

Step 1: Bargaining Power After Communication Breakdown First, we calculate the value of $\hat{p}_{i\ell t}$ when communication breaks down i.e., $\hat{p}_{i\ell t} \equiv E_{\ell t}[i$ wins or j concedes $|$ communication broke down$]$. Applying the Feynman-Kac formula equation,

$$0 \times \hat{p}_{i\ell t} = \nu_{i\ell}[1 - \hat{p}_{i\ell t}] + \nu_{j\ell}[0 - \hat{p}_{i\ell t}] + \sum_{\ell' > \ell} \lambda_{\ell\ell'}[\hat{p}_{i\ell' t} - \hat{p}_{i\ell' t}] + \dot{\hat{p}}_{i\ell t}$$

Since concessions can no longer arrive and players are forced to actively dispute, then the only way for i to win is by attaining a decisive victory. Since the arrival rate of state transitions does not depend on time, then $\dot{\hat{p}}_{i\ell t} = 0$ and $\hat{p}_{i\ell t} = \hat{p}_{i\ell}$ which satisfy that

$$\hat{p}_{i\ell} = \frac{\nu_{i\ell} + \sum_{\ell' > \ell} \lambda_{\ell\ell'} \hat{p}_{i\ell'}}{\nu_{i\ell} + \nu_{j\ell} + \sum_{\ell' > \ell} \lambda_{\ell\ell'}} \tag{34}$$

and when $\ell = n$, then

$$\hat{p}_{in} = \frac{\nu_{in}}{\nu_{i\ell} + \nu_{j\ell}} \tag{35}$$

Note that $\hat{p}_{i\ell}$, for $\ell \in \{1, \dots, n-1\}$, can be derived via backwards induction.

Step 2: Bargaining Power Before Breakdown Next, we derive the expression for bargaining power prior to the line of communication being destroyed. Fix some stable, strategic time $t > 0$ and some state ℓ, the i's bargaining power is $p_{i\ell t} \equiv E_{\ell t}[e^{-r\tau} i \text{ wins or } j \text{ concedes}]$ and the Feynman-Kac formula implies that

$$\begin{aligned} 0 \times p_{i\ell t} = [C_{j\ell t} + \psi_t \nu_{i\ell}][1 - p_{i\ell t}] + [C_{i\ell t} + \psi_t \nu_{j\ell}][0 - p_{i\ell t}] + \phi \sum_i \mu_{i\ell t}[\hat{p}_{i\ell} - p_{i\ell t}] \\ + \sum_{\ell' > \ell} \lambda_{\ell\ell' t}[0(1 - q_{i\ell' t}) + q_{i\ell' t}(1 - q_{j\ell' t}) + q_{i\ell' t} q_{j\ell' t} p_{i\ell' t} - p_{i\ell t}] + \dot{p}_{i\ell t} \end{aligned} \tag{36}$$

The first part notes that i can either wins when j concedes or she attains a decisive victory and, similarly, she loses due to her making concessions or j attaining a decisive victory. Likewise, the communication can breakdown and the surplus attained the state can shift prompting additional concessions.

Regrouping terms, it holds that

$$\begin{aligned} -\left[C_{j\ell t} + \psi_t \nu_{i\ell} + \phi \sum_i \mu_{i\ell t} \hat{p}_{i\ell} + \sum_{\ell' > \ell} \lambda_{\ell\ell' t}[q_{i\ell' t}(1 - q_{j\ell' t}) + q_{i\ell' t} q_{j\ell' t} p_{i\ell' t}]\right] = \\ \dot{p}_{i\ell t} - p_{i\ell t}\left[C_{j\ell t} + \psi_t \nu_{i\ell} + C_{i\ell t} + \psi_t \nu_{j\ell} + \phi \sum_i \mu_{i\ell t} + \sum_{\ell' > \ell} \lambda_{\ell\ell' t}\right] \end{aligned} \tag{37}$$

Adding and subtracting 2ϕ from the right-hand brackets and regrouping terms yields

$$\begin{aligned} -\left[C_{j\ell t} + \psi_t \nu_{i\ell} + \phi \sum_i \mu_{i\ell t} \hat{p}_{i\ell} + \sum_{\ell' > \ell} \lambda_{\ell\ell' t}[q_{i\ell' t}(1 - q_{j\ell' t}) + q_{i\ell' t} q_{j\ell' t} p_{i\ell' t}]\right] = \\ \dot{p}_{i\ell t} - p_{i\ell t}\left[2\phi + \sum_i \left[\psi_t \nu_{i\ell} + \frac{\dot{\mu}_{i\ell t}}{\mu_{i\ell t}}\right] + \sum_{\ell' > \ell} \lambda_{\ell\ell' t}\right] \end{aligned} \tag{38}$$

where the equation above uses the result stating that $\frac{\dot{\mu}_{i\ell t}}{\mu_{i\ell t}} = C_{i\ell t} - \phi(1 - \mu_{i\ell t})$. Using standard arguments, the equation above can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{e^{-2\phi t-\int_0^t \psi_s \sum_i \nu_{i\ell}+\sum_{\ell'>\ell}\lambda_{\ell\ell' s}\mathrm{ds}}p_{i\ell t}}{\prod_i \mu_{i\ell t}}\right] = -\frac{e^{-2\phi t-\int_0^t \psi_s \sum_i \nu_{i\ell}+\sum_{\ell'>\ell}\lambda_{\ell\ell' s}\mathrm{ds}}}{\prod_i \mu_{i\ell t}} \times \left[C_{j\ell t}+\psi_t\nu_{i\ell}+\phi\sum_i \mu_{i\ell t}\hat{p}_{i\ell}+\sum_{\ell'>\ell}\lambda_{\ell\ell' t}[q_{i\ell' t}(1-q_{j\ell' t})+q_{i\ell' t}q_{j\ell' t}p_{i\ell' t}]\right] \quad (39)$$

If one integrates both sides, then one finds that there exists a constant Ψ_ℓ such that

$$\frac{e^{-2\phi t-\int_0^t \psi_s \sum_i \nu_{i\ell}+\psi_s \sum_i \nu_{i\ell}+\sum_{\ell'>\ell}\lambda_{\ell\ell' s}\mathrm{ds}}p_{i\ell t}}{\prod_i \mu_{i\ell t}} = \Psi_\ell - \int_0^t \frac{e^{-2\phi s-\int_0^s \psi_w \sum_i \nu_{i\ell}+\psi_s \sum_i \nu_{i\ell}+\sum_{\ell'>\ell}\lambda_{\ell\ell' w}\mathrm{dw}}}{\prod_i \mu_{i\ell s}} \times \left[C_{j\ell s}+\psi_s\nu_{i\ell}+\phi\sum_i \mu_{i\ell s}\hat{p}_{i\ell}+\sum_{\ell'>\ell}\lambda_{\ell\ell' s}[q_{i\ell' s}(1-q_{j\ell' s})+q_{i\ell' s}q_{j\ell' s}p_{i\ell' s}]\right]\mathrm{ds} \quad (40)$$

Step 3: Solving the Forward Equation To derive Ψ_1, we characterize the discounted probability that i wins once it is certain that both players are obstinate. Note that there exists an earliest time T by which beliefs converge to one if the state does not shift or either player attains a decisive, favorable adjudication. For $t \geq T$, it therefore holds that for each $i, \ell' > 1, \mu_{i1t} = q_{i\ell' t} = 1$ and $C_{i1t} = 0$, so the equation above becomes

$$e^{-2\phi t-\int_0^t \psi_s \sum_i \nu_{i\ell}+\sum_{\ell'>1}\lambda_{\ell 1 s}\mathrm{ds}}\tilde{p}_{i1t} = \tilde{\Psi}_1 - \int_0^t e^{-2\phi s-\int_0^s \psi_w \sum_i \nu_{i\ell}+\sum_{\ell'>1}\lambda_{\ell 1 w}\mathrm{dw}}\left[\psi_s\nu_{i1}+2\phi+\sum_{\ell'>1}\lambda_{\ell 1 s}p_{i\ell' s}\right]\mathrm{ds} \quad (41)$$

and since for each i and ℓ, it holds that $0 \leq \inf_{t>0} p_{i\ell t}$ and $\sup_{t>0} p_{i\ell t} \leq 1$, then the equation above implies that

$$\tilde{\Psi}_1 = \int_0^\infty e^{-2\phi s-\int_0^s \psi_w \sum_i \nu_{i\ell}+\sum_{\ell'>1}\lambda_{1\ell' w}\mathrm{dw}}\left[\psi_s\nu_{i1}+2\phi+\sum_{\ell'>1}\lambda_{\ell 1 s}p_{i\ell' s}\right]\mathrm{ds} \quad (42)$$

In turn, we use this expression to derive an expression for p_{i1t} for $t \geq T$ and

$$\tilde{p}_{i1t} = \int_t^\infty e^{-2\phi(s-t) - \int_t^s \psi_w \sum_i \nu_{i\ell} + \sum_{\ell'>1} \lambda_{1\ell' w} \mathrm{dw}} \left[\psi_s \nu_{i1} + 2\phi + \sum_{\ell'>1} \lambda_{\ell 1 s} p_{i\ell' s} \right] \mathrm{ds} \tag{43}$$

We then impose that at time $T, \tilde{p}_{iT} = p_{iT}$. Plugging this observation into the equation 40 when $\ell = 1$ and $t = T$ yields that

$$\begin{aligned} \Psi_1 = \int_T^\infty & e^{-2\phi s - \int_0^s \psi_w \sum_i \nu_{i1} + \sum_{\ell'>1} \lambda_{1\ell' w} \mathrm{dw}} \left[\psi_s \nu_{i1} + 2\phi + \sum_{\ell'>1} \lambda_{\ell 1 s} p_{i\ell' s} \right] \mathrm{ds} \\ & + \int_0^T \frac{e^{-2\phi s - \int_0^s \psi_w \sum_i \nu_{i1} + \sum_{\ell'>1} \lambda_{1\ell' w} \mathrm{dw}}}{\prod_i \mu_{i1s}} \\ & \times \left[C_{j1s} + \psi_s \nu_{i1} + \phi \sum_i \mu_{i1s} \hat{p}_{i1} + \sum_{\ell'>1} \lambda_{1\ell' s} [q_{i\ell' s}(1 - q_{j\ell' s}) + q_{i\ell' s} q_{j\ell' s} p_{i\ell' s}] \right] \mathrm{ds} \end{aligned} \tag{44}$$

Next, we plug this expression for Ψ_1 back into equation 40 for $t \leq T$, to get that

$$\begin{aligned} p_{i1t} = \prod_i \mu_{i1t} \int_T^\infty & e^{-2\phi(s-t) - \int_t^s \psi_w \sum_i \nu_{i1} + \sum_{\ell'>1} \lambda_{1\ell' w} \mathrm{dw}} \left[\psi_s \nu_{i1} + 2\phi + \sum_{\ell'>1} \lambda_{\ell 1 s} p_{i\ell' s} \right] \mathrm{ds} \\ & + \prod_i \mu_{i1t} \int_t^T \frac{e^{-2\phi(s-t) - \int_t^s \psi_w \sum_i \nu_{i1} + \sum_{\ell'>1} \lambda_{1\ell' w} \mathrm{dw}}}{\prod_i \mu_{i1s}} \\ & \times \left[C_{j1s} + \psi_s \nu_{i1} + \phi \sum_i \mu_{i1s} \hat{p}_{i1} + \sum_{\ell'>1} \lambda_{1\ell' s} [q_{i\ell' s}(1 - q_{j\ell' s}) + q_{i\ell' s} q_{j\ell' s} p_{i\ell' s}] \right] \mathrm{ds} \end{aligned} \tag{45}$$

Step 4: Taking Limits as Frictions Vanish Our object of interest is then p_{1i0} as $(\phi, \bar{c}, \mu, (\lambda_{\ell\ell' t})) \to 0$. First, taking the limit as $(\lambda_{\ell\ell' t}) \to 0$ yields that at time 0

$$\begin{aligned} p_{i10^+} &= \nu_{i1} \prod_i \mu_{i0^+} \int_T^\infty e^{-\sum_i \nu_{i1} \int_0^s \psi_w \mathrm{dw}} \psi_s \mathrm{ds} + \prod_i \mu_{i0^+} \int_0^T \frac{e^{-\sum_i \nu_{i1} \int_0^s \psi_w \mathrm{dw}}}{\prod_i \mu_{i1s}} [C_{j1s} + \psi_s \nu_{i1}] \mathrm{ds} \\ &= \prod_i \mu_{i0^+} \left(\frac{\nu_{i1}}{\nu_{i1} + \nu_{j1}} \right) e^{-\sum_i \nu_{i1} \int_0^T \psi_w \mathrm{dw}} + \prod_i \mu_{i0^+} \int_0^T \frac{e^{-\sum_i \nu_{i1} \int_0^s \psi_w \mathrm{dw}}}{\prod_i \mu_{i1s}} [C_{j1s} + \psi_s \nu_{i1}] \mathrm{ds} \end{aligned} \tag{46}$$

Further note that since $\phi = 0$, then for each player i and time $s > 0$, $\mu_{i1s} = \mu_{i0^+} e^{\int_0^s C_{i1w} \mathrm{dw}}$. Plugging this observation into the equation above implies that

$$p_{i10^+} = \prod_i \mu_{i0^+} \left(\frac{\nu_{i1}}{\nu_{i1} + \nu_{j1}} \right) e^{-\sum_i \nu_{i1} \int_0^T \psi_w \mathrm{dw}} + \int_0^T e^{-\int_0^s \sum_i C_{i1s} + \nu_{i1} \psi_w \mathrm{dw}} [C_{j1s} + \psi_s \nu_{i1}] \mathrm{ds} \tag{47}$$

Next, observe that it is further the case that as $(\phi, \bar{c}, \mu, (\lambda_{\ell\ell' t})) \to 0$, $C_{j1s} = \bar{c} + \psi_s(c_{i1} - \nu_{j1})$, so the equation above further simplifies into

$$p_{i10^+} = \prod_i \mu_{i0^+} \left(\frac{\nu_{i1}}{\nu_{i1} + \nu_{j1}} \right) e^{-\sum_i \nu_{i1} \int_0^T \psi_w \mathrm{dw}} + \int_0^T e^{-2\bar{c}t + \sum_i c_{i1} \int_0^s \psi_w \mathrm{dw}} [\bar{c} + \psi_s c_{i1}] \mathrm{ds} \tag{48}$$

We then take the limit as $\bar{c} \to 0$ and find that

$$p_{i10^+} \quad = \quad \prod_i \mu_{i0^+} \left(\frac{\nu_{i1}}{\nu_{i1} + \nu_{j1}} \right) e^{-\sum_i \nu_{i1} \int_0^T \psi_w \mathrm{dw}} \quad + \quad c_{i1} \int_0^T e^{-\sum_i c_{i1} \int_0^s \psi_w \mathrm{dw}} \psi_s \mathrm{ds} \tag{49}$$

We can then rewrite the integral by multiplying by one as

$$p_{i10^+} = \prod_i \mu_{i0^+} \left(\frac{\nu_{i1}}{\nu_{i1} + \nu_{j1}} \right) e^{-\sum_i \nu_{i1} \int_0^T \psi_w \mathrm{dw}} - \frac{c_{i1}}{c_{i1} + c_{j1}} \int_0^T - e^{-\sum_i c_{i1} \int_0^s \psi_w \mathrm{dw}} [\psi_s \sum_i c_{is}] \mathrm{ds} \tag{50}$$

Next, we can note that $-e^{-\sum_i c_{i1} \int_0^s \psi_w \mathrm{dw}} [\psi_s \sum_i c_{is}]$ is just the time derivative of $e^{-\sum_i c_{i1} \int_0^s \psi_w \mathrm{dw}}$, so the equation above becomes

$$p_{i10^+} = \prod_i \mu_{i0^+} \left(\frac{\nu_{i1}}{\nu_{i1} + \nu_{j1}} \right) e^{-\sum_i \nu_{i1} \int_0^T \psi_w \mathrm{dw}} - \frac{c_{i1}}{c_{i1} + c_{j1}} \int_0^T \frac{\mathrm{d}}{\mathrm{ds}} e^{-\sum_i c_{i1} \int_0^s \psi_w \mathrm{dw}} \mathrm{ds} \tag{51}$$

We then observe that one can directly solve the integral expression above to yield that

$$p_{i10^+} = \prod_i \mu_{i0^+} \left(\frac{\nu_{i1}}{\nu_{i1} + \nu_{j1}} \right) e^{-\sum_i \nu_{i1} \int_0^T \psi_w \mathrm{dw}} + \left(\frac{c_{i1}}{c_{i1} + c_{j1}} \right) \left(1 - e^{-\sum_i \nu_{i1} \int_0^T \psi_w \mathrm{dw}}\right) \tag{52}$$

Step 5: Closed-Form Expression and Interpretation Lastly, observe that since beliefs lie between 0 and 1 and (at most) one player concedes from the outset with a positive probability, then $\prod_i \mu_{i0^+} \leq \mu$. It is also the case that as $\mu \to 0, T \to \infty$. Consequently, as $\mu \to 0, \prod_i \mu_{i0^+} \to 0$ and the limiting probability of a concession becomes p_i where

$$p_i = \left(\frac{c_{i1}}{c_{i1} + c_{j1}} \right) \left(1 - e^{-\int_0^\infty \psi_{t|\ell=1} \mathrm{dt} \sum_i \nu_{i1}}\right) \tag{53}$$

This concludes the proof.

2 Robustness Check: De-escalation in the Labor Strikes context

To assess whether our findings extend beyond litigation, we conduct a secondary test in the domain of labor disputes. This allows us to evaluate whether the observed dynamics reflect a general mechanism of procedural de-escalation, rather than a litigation-specific pathology.

We examine the effect of mass picketing restrictions—state-level policies that limit the number of workers allowed to picket—on strike duration. These laws, adopted decades before most modern strikes, were primarily enacted in Southern and right-to-work states. As such, they influence disputes not through direct legal intervention during a strike, but by shaping the conditions under which labor actions unfold. Consistent with the model's predictions, we find that picketing restrictions are associated with longer strike durations. Figure 1 provides descriptive evidence consistent with this mechanism.

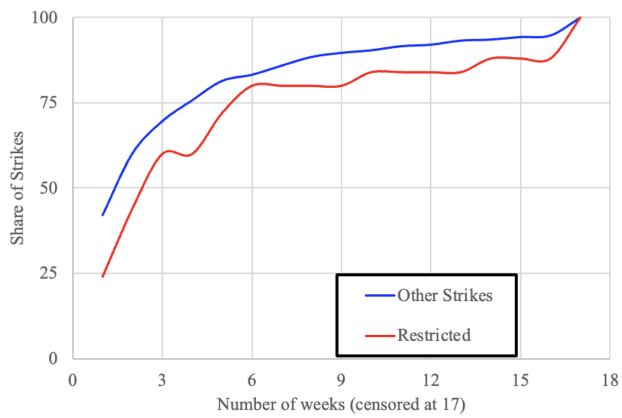


Figure 1: Cumulative Distribution function for the total number of weeks that strikes lasted by whether or not a strike took place in a jurisdiction with mass picketing restrictions.
Source. Author's calculation from data from the US Bureau of Labor Statistics.

2.0.1 Institutional Design and Theoretical Mapping

Mass picketing laws enacted between 1940-1980 provide an empirical laboratory for testing the SDP through a clean two-stage identification strategy. These statutes impose uniform restrictions detailing how strikes can be conducted e.g., imposing location restrictions, capping the number of picketers, et cetera. They implicitly ensure full de-escalation from the outset and have been institutionalized decades before the strikes observed in our sample.

2.0.2 Econometric Framework

Our identification exploits cross-state, within-time variation in statutory constraint intensity:

$$\text{First stage:} \quad \text{IdleShare}_{ist} = \alpha + \beta \text{LawAge}_{st} + \gamma X_{ist} + \delta_s + \lambda_t + \epsilon_{ist} \tag{54}$$

$$\text{Second stage:} \quad \ln(\text{StrikeDuration}_{ist}) = \theta \widehat{\text{IdleShare}}_{ist} + \gamma X_{ist} + \delta_s + \lambda_t + \nu_{ist} \tag{55}$$

where IdleShare_{ist} captures the proportion of strike time during which workers are entirely idle—i.e., not substituting with alternative tactics like slowdowns or pressure campaigns. LawAge_{st} measures years since enactment, proxying accumulated institutionalization: older statutes develop clear enforcement routines, judicial precedents, and shared expectations among actors. The exclusion restriction requires that law age affects duration only through tactical choice between picketing and work stoppage.

2.0.3 Identification Strategy

Exclusion Restriction Defense: Our identification leverages three features. First, a temporal gap of 20-70 years between enactment and observed strikes ensures laws arose from broader labor regulation waves rather than dispute-specific responses. Second, while unions have alternative tactics, picketing uniquely imposes immediate, visible production costs—removing it forces costly substitutes. Third, to address concerns that laws proxy for anti-union sentiment, we include state fixed effects and control for unionization rates, right-to-work laws, and other policies. These controls absorb both the enduring political climate and the slow-moving institutional hostility toward organized labor. Results are robust to excluding Southern states and controlling for Democratic governors.

First-Stage Strength: The F-statistic exceeds 18. Law age robustly predicts increased idle share ($\beta = 0.08$ percentage points per year, t = 4.3). By restricting picketing, these laws strip unions of intermediate escalation tools—forcing disputes into binary, high-cost confrontations. This drives idle share mechanically—not from higher dispute intensity, but from constrained tactical options.

2.1 Legal Context: Mass Picketing laws restrictions

Several southern and right-to-work (RTW) states enacted statutes between the 1940s and 1970s that sharply curtailed labor picketing during private-sector strikes. These mass picketing laws, though originally justified as public safety measures, functionally operate as a de-escalation mechanisms: they suppress visible forms of disruption without conditioning on actor behavior. In the language of the model, they impose full de-escalation i.e., $\psi_t = 0$ at each time $t \geq 0$. Unlike injunctions granted after specific actions, these restrictions apply *a* *priori*—limiting crowd size, noise, location, and access regardless of the underlying conduct. We argue that these statutes are analogous to the early de-escalation policies that can cause the SPT mechanism.

These laws differ from right-to-work statutes in an important way: while RTW weakens long-run union capacity, picketing restrictions dampen short-run strategic leverage. By muting the reputational, economic, and signaling effects of protest, they blunt the mechanisms by which strikes generate urgency. Employers face fewer costs from delay, while unions lose key tools for mobilizing public sympathy or internal discipline. In effect, these laws induce early-stage de-escalation independent of case-specific behavior—precisely the institutional design feature that activates the SPT.

We focus on six states with the most restrictive and durable mass picketing provisions:

i. **Georgia (1963)** — Ga. Code Ann. § 34-6-5 prohibits "mass picketing" that obstructs entrance to a place of business. Courts routinely issue injunctions on this basis, limiting the number of picketers per entrance to 2–4 individuals.

ii. **Alabama (1953)** — Ala. Code § 25-1-15 criminalizes picketing that interferes with ingress or egress. Enforcement has relied heavily on local courts, which can issue restraining orders without notice.

iii. **North Carolina (1979)** — N.C. Gen. Stat. § 95-79(b) prohibits "secondary boycott activity" and large-scale picketing, especially where it interferes with business access. The statute has been used to limit coordination across sites and industries.

iv. **South Carolina (1954)** — S.C. Code Ann. § 41-7-90 bars picketing near private residences or business entrances if deemed "coercive." Though vague, courts have interpreted this to justify preemptive restrictions.

v. **Mississippi (1952)** — Miss. Code Ann. § 71-1-47 bans more than three individuals from picketing any private workplace. Violations are treated as criminal misdemeanors and carry civil liability.

vi. **Texas (1947)** — Tex. Labor Code § 101.001–101.106 authorizes employers to seek injunctive relief against picketing that "intimidates or obstructs," including via verbal confrontation or signage deemed threatening.

Each law codifies a structural ceiling on disruptive capacity. Because these ceilings are fixed by statute and enforced procedurally, they generate variation in the *de jure* level of protest permissible at the onset of dispute. This exogenous constraint on visibility and disruption maps directly to our theoretical construct of de-escalation: relief that arrives mechanically and indiscriminately, suppressing strategic pressure without resolving the underlying dispute.

2.2 Empirical Strategy and Instruments

We analyze US private-sector strikes from 1985 to 2023, focusing on events with valid start and end dates. The outcome is the natural log of strike duration. Our primary goal is to estimate how union-side cost exposure—and its distortion through legal and institutional structures—affects the timing of settlement. We operationalize cost exposure using the share of strike time during which workers were fully idle, a proxy for tactical endurance. Because stronger unions are more capable of sustaining prolonged inactivity, this idle share is endogenous to unobserved bargaining strength.

To address this, we instrument for idle share using three state-level variables: (1) years since adoption of right-to-work (RTW) laws, (2) private-sector unionization rates in 1964, and (3) years since passage of laws banning captive-audience meetings. These instruments capture long-run variation in union-side financial and legal capacity. Crucially, none directly affect procedural bargaining rules or strike timing—only the underlying ability of unions to bear costs. While each instrument carries potential exclusion risks (e.g., ideology, industrial composition), we mitigate these concerns by including fixed effects for state, industry, and union identity. These absorb persistent differences in labor-market structure and institutional context.

Our identification strategy exploits long-standing policy variation enacted decades before the observed strikes, providing credible exogenous variation in union cost-bearing capacity. Right-to-work laws and captive-audience regulations were adopted for broad political reasons unrelated to specific labor disputes in our sample, satisfying standard exclusion restrictions. The temporal separation between policy adoption and observed strikes eliminates concerns about endogenous policy responses to contemporaneous bargaining dynamics. Our estimates demonstrate that reduced cost endurance systematically delays concession timing, consistent with the theory's core prediction that institutional constraints on strategic pressure prolong disputes.

Key instrument capturing SDP We also analyze the effect of mass picketing laws as a direct test of the SDP. Picketing restrictions vary across states, but the vast majority were enacted well before our strike data begin. The median picketing law in force was adopted between 1950 and 1980—decades prior to any of the disputes in our sample. These laws emerged from general labor regulatory reform, not in response to specific disputes, and are unlikely to reflect targeted employer pressure or reactive policymaking.

Similarly, RTW laws were mostly enacted by conservative legislatures long before the strikes observed here. While RTW laws primarily reduce baseline union leverage—potentially aligning with the logic of Strategic Concession Deferral—picketing laws function as procedural constraints on strike intensity, independent of actor behavior. They prohibit tactics such as secondary pickets and slowdowns, and serve as formal analogs to institutional de-escalation policies in the model.

By jointly analyzing these dimensions—structural union capacity (via instruments) and procedural de-escalation (via picketing laws)—we empirically isolate two mechanisms of interest: how weakened cost endurance reshapes concession timing (SCD), and how blanket relief policies alter the duration of dispute (SDP).

2.3 Placebo Test: Instruments and Exclusion Validity

To test the exclusion restriction, we regress log strike duration directly on the instruments, controlling for the same fixed effects. Table 1 shows that three of the four variables—RTW, picketing restrictions, and historical unionization—have statistically insignificant coefficients. Only the labor protection index enters significantly, and is therefore excluded from our IV models.

These results support the claim that the retained instruments affect strike duration only through short-run union-side cost dynamics. Their null effect on duration, once controls are included, reinforces their interpretation as valid, exogenous sources of endurance variation.

Table 1: Placebo Test: Instruments and Strike Duration (with Fixed Effects)

	Coefficient	Std. Error	p-value
Right-to-Work Law (RTW)	-0.000	0.009	0.985
Mass Picketing Restriction	-0.009	0.013	0.490
Unionization Rate	-0.021	0.061	0.734
No mandatory anti-unionization meetings	0.009^{**}	0.004	0.026
State Fixed Effects	Yes		
Union Fixed Effects	Yes		
Industry Fixed Effects	Yes		
Observations	691		
R-squared	0.963		
Adjusted R-squared	0.947		

$^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 2: Instrumental Variables Regression: Log Duration of Strikes

	Log of Duration (in days)		
	(1)	(2)	(3)
Log of the share of time spent idle	-1.805^{***}	-1.817^{***}	-1.734^{***}
Log of number of striking workers	0.012	0.012	0.015^{*}
State Fixed Effects	Yes	Yes	Yes
Industry Fixed Effects	No	Yes	Yes
Union Fixed Effects	No	No	Yes
Constant	0.092	0.609	−0.977
Observations	692	692	691
R-squared	0.991	0.991	0.992
Weak IV Tests (p-values)			
CLR	0.033	0.038	0.009
K	0.011	0.011	0.003
J (Over-ID test)	0.513	0.576	0.752
K–J	0.013	0.014	0.004
AR	0.049	0.056	0.024
Wald	0.000	0.000	0.000
First Stage F-tests (p-values)			
Sanderson-Windmeijer	0.033	0.0414	0.025

Notes: Standard errors in parentheses. FE = Fixed Effects.
$^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

2.4 Main Results and Diagnostics

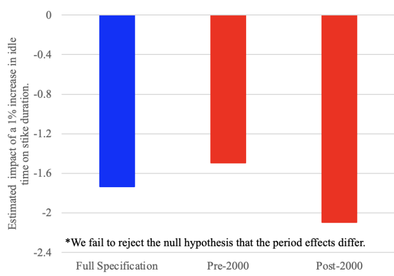
Table 2 reports second-stage estimates. A one-unit increase in instrumented idle share increases log strike duration by 1.73–1.81 across specifications. At the sample mean (idle share ≈ 0.74), this implies a 15–20% increase in strike length—adding 5-7 days to the mean 35-day strike.

IV diagnostics further confirm strong identification. First-stage F-statistics exceed conventional thresholds across specifications. Weak instrument tests (CLR, AR, Wald) reject with $p < 0.01$ in most models. Hansen J-statistics show no overidentification concerns ($p > 0.5$ throughout). The instruments are statistically strong and pass standard validity tests.

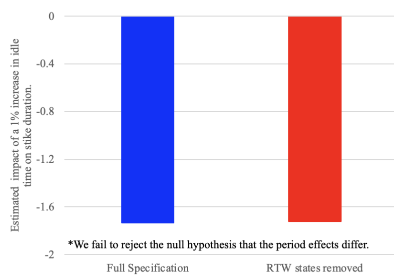
2.5 Robustness and Mechanism Validation

We conduct three additional robustness checks to validate our identification strategy and test core mechanism predictions. These checks address concerns about temporal shifts in labor relations, bundled policy effects, and sector-specific dynamics.

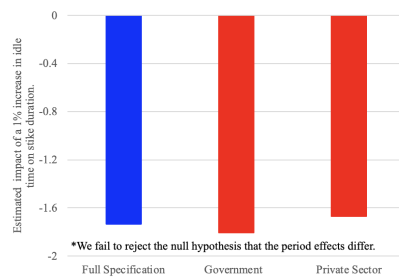
Figure 2 presents results across three dimensions of variation. Panel A examines temporal stability by splitting the sample at 2000, when union density and bargaining power experienced a notable decline. Panel B tests whether effects persist when excluding right-to-work states entirely, isolating picketing restrictions from broader anti-union policy bundles. Panel C compares government and private sector strikes to test whether institutional dynamics vary across bargaining contexts.



(a) Pre/Post-2000 Split



(b) RTW States Removed



(c) Government vs. Private

Figure 2: Robustness checks for labor application. All three checks confirm our core finding that reduced cost-bearing capacity delays settlement, with effects intensifying as union bargaining power weakened.

Panel (a) shows a clear, trend effect. The effect intensifies substantially post-2000, with coefficients rising from -1.5 to -2.1. This pattern aligns with SDP predictions: as baseline union capacity weakened over time, cost-suppression policies gained greater strategic bite. Weaker unions became more vulnerable to institutional constraints on pressure tactics, amplifying delays.

In Panel (b), excluding all right-to-work states preserves the core effect (-1.7), ruling out concerns that picketing restrictions merely proxy for broader anti-union sentiment. This confirms that procedural constraints on strike tactics operate independently of structural union-weakening policies. Lastly,

Panel (c) finds that there is not economic or statistically significant difference in the impact by sector (government versus private), supporting institutional rather than sector-specific mechanisms. SDP operates through procedural timing constraints rather than industry-specific bargaining dynamics or cost structures.

These robustness checks collectively demonstrate that our findings reflect institutional timing effects rather than confounding from union decline, bundled policies, or sector-specific factors. The post-2000 intensification particularly supports the mechanism: weaker bargaining positions amplify the strategic consequences of cost-suppression policies, exactly as the model predicts.

2.6 Intensive Margin: Cost Endurance and Strike Duration

The labor application provides a clean test of the SDP's intensive margin effects. Our instrumental variables estimates show that a 1% increase in idle share is associated with a reduction in strike duration by 1.73-1.83%.To assess the effect's economic significance, we simulate the counterfactual where all strikes achieve complete work stoppages (idle share = 1). Using our most conservative IV estimate, with full controls, this predicts average strike duration would fall from 35 days to 20 days: a 45% reduction. This 2 week duration reduction represents the full potential of unrestricted cost-imposing capacity. Real-world constraints that reduce idle share below this maximum directly translate into prolonged disputes through the channels our model identifies.