LECTURE 5

JD R-M

THIS WEEK

- Review Standard Monopoly Pricing
- Interpret results (twofold)
- Does picking the quantity matter?
- Finite Value Case
- Price discrimination
- Perfect Price Discrimination
- Limits of Price Discrimination (Bergmann et al (2015))

LAST CLASS

- We have a seller with a demand function $D(.) = 1 - F(.)$ and a marginal cost of $c \geq 0$
- His problem reduced to picking a price p solving

$$(1)\ \pi^* = \max_{p \in \Re_+} (p - c)D(p) = \max_{p \in \Re_+} (p - c)[1 - F(p)]$$

- Optimal price satisfies

$$(2)\ p^* = c + \left[\frac{D(p^*)}{-D'(p^*)}\right] = c + \frac{1 - F(p^*)}{f(p^*)}$$

- Equation (2) is profoundly vague by itself…

INTERPRETATION

- Remember elasticity of demand

$$(3)\ \forall p \geq 0, \epsilon(p) = -p\frac{d}{dp}\ln D(p) = p\left[\frac{-D'(p)}{D(p)}\right]$$

- It captures the percentage change in the demand that changes given some change in the price.
- If we plug expression (3) into (2), it holds that

$$(4)\ p^* = c + \frac{p^*}{\epsilon(p^*)}$$

- *Re-organizing*

$$(5)\ p^* = \underbrace{c}_{Production\ Cost} \quad \underbrace{\frac{\epsilon(p^*)}{1-\epsilon(p^*)}}_{Surplus\ Extraction}$$

PICKING THE QUANTITY

- It is clear to see that the monopolist could pick the quantity sold instead $q \in [0,1]$
- By market clearing, the resulting price satisfies $q = 1 - F(p)$, so $P(q) = F^{-1}(1-q)$
- The seller's problem becomes

$$(6)\ \pi^* = \max_{q \in [0,1]} q[P(q) - c]$$

- The optimum is interior and solves the FOC

$$(7)\ c = P(q^*) + q^* P'(q^*) = p^* - \frac{q^*}{f(p^*)} = p^* - \frac{1 - F(p^*)}{f(p^*)} = \hat{v}(p^*)$$

- Punchline: IN THE MONOPOLY CASE, picking supply and quantity is the same thing (modulus technical details).
- Picture: in class…

NUMERICAL EXAMPLE

- Assume $\forall p \in \Re_+, F(p) = 1 - e^{-\frac{p}{\lambda}}, D(p) = 1 - F(p) = e^{-\frac{p}{\lambda}}$ for $\lambda > 0$ and marginal cost is $c > 0$
- Note that $\forall p \geq 0, \epsilon(p) = \frac{p}{\lambda}, \frac{\epsilon(p)}{\epsilon(p)-1} = \frac{p}{p-\lambda}$
- Seller's problem is then to choose $p \geq 0$ to solve

$$\pi_s^* = \max_{p \geq 0}(p - c)e^{-\frac{p}{\lambda}}$$

- *The* foc is interior and satisfies

$$e^{-\frac{p}{\lambda}} - (p - c)\frac{e^{-\frac{p}{\lambda}}}{\lambda} = 0$$

- *Thus,* the price satisfies a decomposition

$$p^* = c + \lambda = c\left[\frac{c + \lambda}{c}\right]$$

A SIMPLER EXAMPLE (EXAMPLE *)

- Suppose that valuations $\theta \in \{1,2,3\}$
- $\Pr(\theta = 1) = \Pr(\theta = 2) = \Pr(\theta = 3) = \frac{1}{3}$
- Market operates as before
- Marginal cost $c = 0$
- Then seller might as well pick a price $p \in \{1,2,3\}$

REVENUES ARE

- Profits can then be written as

$$\pi(p) = \begin{cases} 1\,\Pr(\theta \geq 1) = 1 \; if \; p = 1 \\ 2\,\Pr(\theta \geq 2) = 2\left(\frac{2}{3}\right) = \frac{4}{3} \; if \; p = 2 \\ 3\,\Pr(\theta \geq 3) = 3\left(\frac{1}{3}\right) = 1 \; if \; p = 3 \end{cases}$$

- Optimal price is $p^* = 2$ and profits equal to $\pi^* = \frac{4}{3}$
- Consumer Surplus is $u^* = \frac{1}{3}$
- I will return to this problem.

PRICE DISCRIMINATION

- If the seller can distinguish between consumers, he can offer charge each buyer a different price
- This behavior, uninspiringly, is called “price discrimination”.
- Q: But does it occur in the real world and if so how?
- A: A lot. Ways sellers price discriminate include:

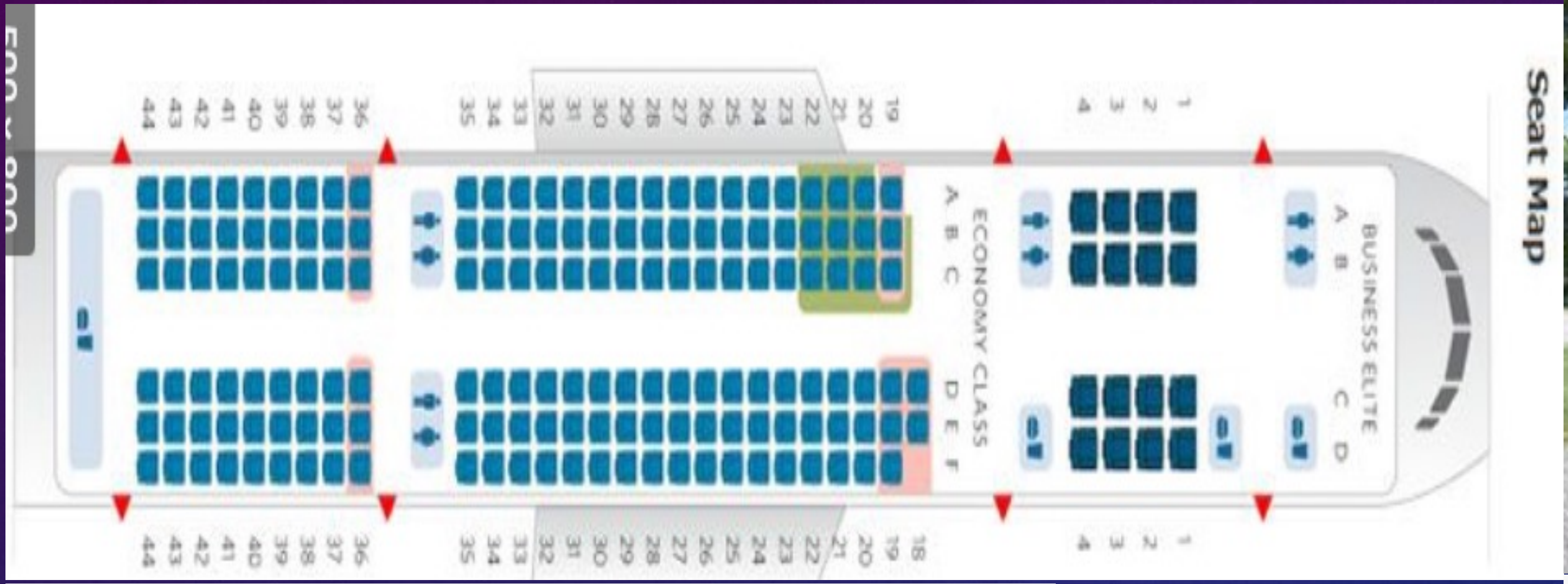
i. demographics (sex, race, ethnicity)

ii. descriptors (wealth, education, political affiliation, profession)

iii. Geography

DEMOGRAPHIC DISCRIMINATION

CLASS



Seat Map
BUSINESS ELITE
ECONOMY CLASS

BLUE MOON

Coors
LIGHT

GEOGRAPHIC

- Airport food and restaurants (Think buying a water bottle at the airport!)
- Malls
- Car sellers
- Regional Differences (Back home every dessert had Nutella a couple years ago…)

POLITICAL AFFILIATION

- A rationalization for brands to get involved in politics is to segment the market
- Remember your demographics may be exogenously determined, BUT political affiliation is not.
- By focusing on a given market in which you know the demand with more granularity, you can extract greater rents from consumers.

PERFECT PRICE DISCRIMINATION

- Once we established how a monopolist prices goods when he cannot price discriminate, what happens in the opposite extreme?
- What if the seller knows precisely how much to charge each possible agent?
- The next section discusses the environment and equilibrium in such setting.

PERFECT PRICE DISCRIMINATION (A SILLY MARKET)

- Players: 1 seller, unit mass continuum of consumers
- Actions: Seller picks prices p_i, consumers buy 1 or 0 units
- Payoffs: As before
- Timing:

1. Nature picks a valuation for each buyer i, $\theta_i \in \Theta \subset \Re_+$ and announces it to the seller and i
2. Seller posts prices (p_i)
3. Buyers, simultaneously, make purchase decisions.

EQUILIBRIUM

- A seller strategy is a function $p: \Theta \to [0,1], p(\theta)$ price charged to buyer w/ valuation θ
- A buyer strategy is a function $b: \Theta \to \Theta, b(\theta)$ is the purchase decision of a
- An equilibrium is a pair of functions $p: \Theta \to \Theta, b: \Theta \to \{0,1\}$ satisfying the standard optimality conditions
- Outcome is: Seller picks $\forall \theta \in \Theta, p(\theta) = \theta, b(\theta) = 1.$
- Seller profits: $\pi_{pc}^* = E[\theta]$
- Consumer surplus: $u^* = 0.$
- When the seller has more information about his consumer than in the classic setting above, he can always at least do as well as he used to do before having such information
- Because he can always choose to treat all buyers equivalently!!
- Thus, no matter what additional information the seller receives, he can always make π_S^*: monopoly prices attainable by a single price.

NUMERICAL EXAMPLE, BACK TO EXAMPLE *

- A useful case exercise is to calculate profits and consumer surplus, in the example, *
- Consumer surplus is $u^* = 0$
- Profits are $\pi_{pd}^* = 1\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right) = \frac{6}{3} = 2.$
- Q: Could the seller attain higher profits that $\pi^* \geq \pi_{pd}^*$?
- A: No, since buyers must be willing to accept the monopolist’s price.

FACTS WE NOW KNOW…

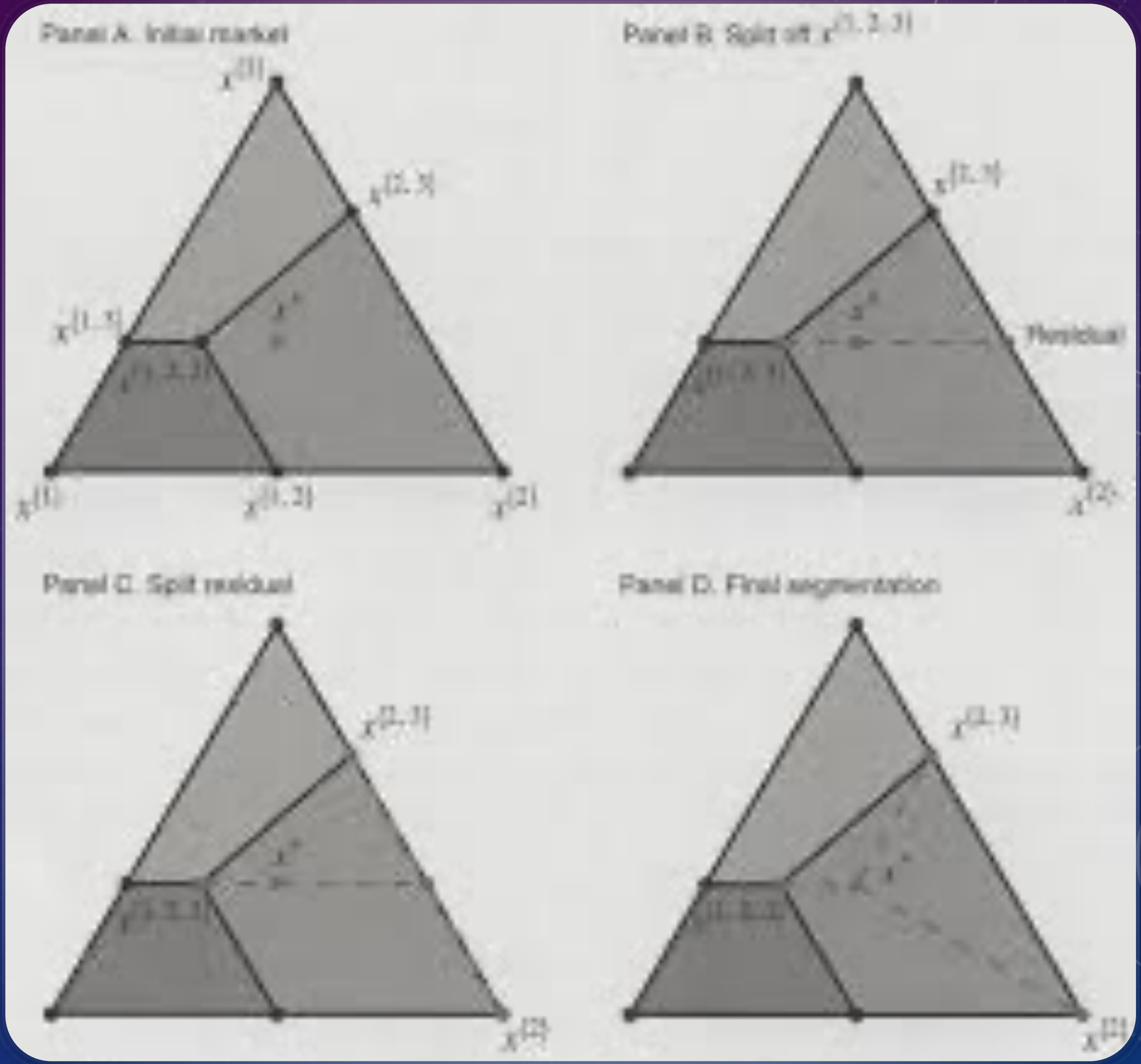
- Profits cannot be higher than in the perfect price discrimination case and we know the following facts

1. Seller Profits cannot fall below profits without price discrimination, so $\pi_S^* \leq \pi^*$,
2. Consumer surplus is non-negative, so $u^* \geq 0$,
3. Sum of total expected valuation $\pi^* + u^* \leq E[\theta]$

- Q: With these facts in mind, could price discrimination allow any combination of profits π^* and consumer surplus u^* so long as they respect the three conditions above?
- A: Yes!

BERGEMANN ET AL (2015)

- These authors established this question
- Unfortunately, their result is HARD.
- I will illustrate their results, from now on, only with example *.



MARKET FRAGMENTATION

- Assume that the set of valuations is finite $\Theta = (\theta_i)_{i=1}^{n} \subset \Re_+$ such that for each I,j where $i < j$, it holds $\theta_i < \theta_j$
- In our case $\Theta = \{1,2,3\}$
- A market is a distribution over valuation or a vector $\lambda = (\lambda_i)_{i=1}^{n} \in \Re_+^n$, such that $\forall i, \lambda_i = \Pr(\theta = \theta_i), \sum_{i=1}^{n} \lambda_i = 1$.
- A *market fragmentation* is a finite collection of markets and proportion of each $\left(\left(\lambda_i^j\right)_{i=1}^{n}, \beta_j\right)_{j=1}^{m}$ such that for each j, $\left(\lambda_i^j\right)_{i=1}^{n}$ is a market, $\beta_j \geq 0$ is a proportion of a whole market belonging to the submarket in question, and $\forall i \in \{1,2,\cdots,n\}$ $\sum_{j=1}^{m} \beta_j \lambda_i^j = \lambda_i, \sum_{j=1}^{m} \beta_j = 1$
- Q: What does this mean?
- A: A market is a demand function and price discrimination can be modeled by arbitrarily grouping consumers into smaller markets.

BERGEMANN ET AL

- These authors show that for every pair (u^*, π^*) satisfying $1.-3.$, there exists a market fragmentation from whom the split of trade surplus follows the quantity discussed.
- I will focus on the market fragmentation maximizing u^* in example*.

WHAT SEGMENTATION WORKS FOR EXAMPLE *

- Initial market is $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
- Consider splitting the market into three parts

Market	λ_1	λ_2	λ_3	Equilibrium prices	Profits by market	Consumer Surplus	Share of Total Population
Poor	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	1	1	$\frac{1}{6} + 2\left(\frac{1}{3}\right) = \frac{5}{6}$	$\frac{2}{3}$
Middle Class	0	1	0	2	2	0	$\frac{1}{6}$
Wealthy	0	$\frac{1}{3}$	$\frac{2}{3}$	2	2	$\frac{2}{3}$	$\frac{1}{6}$
Aggregate	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		$\frac{4}{3}$	$\frac{2}{3}$	1

HOW TO MINIMIZE CONSUMER SURPLUS

Market	λ_1	λ_2	λ_3	Equilibrium prices	Profits by market	Consumer Surplus	Share of Total Population
Poor	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	2	$\frac{3}{2}$	0	$\frac{2}{3}$
Middle Class	$\frac{1}{2}$	0	$\frac{1}{2}$	3	$\frac{3}{2}$	0	$\frac{1}{6}$
Wealthy	$\frac{1}{2}$	0	$\frac{1}{2}$	3	$\frac{3}{2}$	0	$\frac{1}{6}$
Aggregate	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		$\frac{3}{2}$	0	1

IN GENERAL: POLICY PRESCRIPTION

- One can, at best, give the monopoly sufficient information to keep him indifferent between price discriminating and uniform pricing
- Help him identify the “poor” (i.e. those with low valuations) and group them in a single market
- Move the rest into markets in which the seller would weakly prefer to trade with everyone
- Punchline 1: to maximize consumer surplus, trade high rents per trade with few consumers for lower rents while trading with more consumers
- Punchline 2: Price discrimination need not be bad for consumers, BUT it depends on what information the seller observes
- Punchline 3: Maximizing consumer rents requires segregating the low valuation individuals and re-combining the rest

NEXT TIME

- Auctions!!