LECTURE 10---SECOND PART

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FOR TODAY

i. Bertrand Competition ("Price Wars")

EXAMPLES

1. Insurance Providers
2. Walmart Entering a new city in the early 2000s
3. ETFs
4. Game Consoles

ENVIRONMENT

- Agents: Unit mass continuum of consumers and $n \in \{2,3,\cdots\}$ producers
- Actions:

i. Consumers: each consumer can choose to buy $\mathrm{x} = \left(x_j\right)_{j=1}^{n} \in \{0,1\}^n$ and it holds that $\sum_{j=1}^{n} x_j \leq 1$.

ii. Producers: each producer j chooses a price $p_j \in [0,1]$ to charge for his good

- Payoffs:

i. Consumers: If a consumer buys x units of the goods, his returns become $\sum_{j=1}^{n} x_j\left[\theta - p_j\right]$

ii. Producers: If a producer charges $p_j \in [0,1]$ and a share of consumers $m_j \in [0,1]$ buy, then $\pi_j\left(p_j, m_j\right) = \left(p_j - c_j\right)m_j, c_j > 0$

- Technical Conditions:

i. For each consumer, $\forall\theta \in [0,1], F(\theta) = \theta$ and valuations are drawn pairwise independently

ii. For each pair of producers $i, j\ , i > j, 0 \leq c_j \leq c_i \leq 1$

TIMING

i. Nature first draws values for each consumer and privately informs each consumer of their valuation

ii. Producers, simultaneously, post prices $p = (p_j)$

iii. Consumers make their purchase decision

iv. Market closes.

STRATEGIES

- For each producer j, a strategy is a price $p_j \in [0,1]$
- A buyer strategy is a function $b\colon [0,1]^{n+1} \to \left\{ x = \left(x_j\right)_{j=1}^{n} \in \{0,1\}^n \,\middle|\, \sum_{j=1}^{n} x_j \leq 1 \right\}$

CONSUMER PROBLEM

- Given some valuation θ and prices $p = \left(p_j\right)$, seller solves

$$(1)\ CS\ (\theta, p) = \max_{x=\left(x_j\right)_{j=1}^{n} \in \{0,1\}^n} \sum_{j=1}^{n} x_j \left[\theta - p_j\right]$$

PRODUCER PROBLEM

- Conjecturing a demand for his good j $b_j(.,p_j,p_{-j})$ and prices charges by other producers $p_{-j} \in \Re^{n-1}$, Producer picks a price $p_j \in \Re$ solving

$$(2j)\ \pi_j(p_{-j},b_j) = \max_{p_j\in\Re}(p_j - c_j)E\left[b_j(\theta,p_j,p_{-j})\right]$$

EQUILIBRIUM

- An Equilibrium is a tuple $\sigma = \left(p = (p_j)_{j=1}^{n}, b\right)$ such that

i. Given σ, each producer j pick a price p_j solving $(2j)$

ii. Given each p and valuation $\theta, b(\theta, p)$ solves (1)

iii. For each producer j, markets clear.

CHARACTERIZATION 1

- We, again, use backwards induction
- Given some price vector p and a valuation θ a consumer buys good j iff

1. $\theta - p_j \geq 0$
2. $\forall i \neq j, \theta - p_j \geq \theta - p_i$ or that $p_i \geq p_j$

- Demand for each good j is then

$$(3)\ b_j(\theta, p) = \begin{cases} 1 \quad if \ \ p_j \leq \max\left\{\min_{i \neq j} p_i \, , \theta\right\} \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

CHARACTERIZATION 2

- A seller j conjectures prices $p_{-j} = (p_i)_{i \neq j} \in [0,1]^{n-1}$ and he faces two options,
- Option 1: he may pick a price $p_j \geq \min_{i \neq j} p_i \equiv p_j^-$ and net 0 profits
- Option 2: Pick a price price $p_j \leq \min_{i \neq j} p_i$ solving

$$(4)\ \pi_j(p_{-j}) = \max_{p_j \in [0, p_j^-]} (p_j - c_j)(1 - p_j)$$

CHARACTERIZATION 3

- The Lagrange equation is then

$$(5)\ \mathcal{L}(p_j, \lambda) = (p_j - c_j)(1 - p_j) + \lambda_+(p_j^- - p_j) + \lambda_0 p_j, \lambda_+, \lambda_0 \geq 0$$

And the foc and slackness conditions imply that

$$(6) \lambda_0 - \lambda_+ + 1 + c_j - 2p_j = 0, \lambda_+(p_j^- - p_j) = \lambda_0 p_j = 0$$

CHARACTERIZATION 4

- Note that if $p_j^- \leq c_j$, seller j is better of not competing at all; otherwise, his optimal price satisfies the optimality conditions.
- These conditions imply a best response function

$$(7)\ p_j(p_{-j}) = \begin{cases} \min\left\{\frac{1+c_j}{2}, \min_{i\neq j} p_i\right\}\ if\, c_j < \min_{i\neq j} p_i \\ \left[\min_{i\neq j} p_i, 1\right]\ if\ c_j \geq \min_{i\neq j} p_i \end{cases}$$

CHARACTERIZATION 5: SYMMETRIC SELLERS

- Let us now consider the case where all sellers have the same cost $c \in (0,1)$
- I claim that for each pair of sellers $i, j, p_i = p_j = c$

i. First note that buyers would only charge price $p_j \in [c, 1]$

ii. Second, if $\min_{i \neq j} p_i > c$, then the seller nets no profits from charging $p_j \geq \min_{i \neq j} p_i$ and

$$\pi_j(p_{-j}) = \begin{cases} \left(\frac{1-c}{2}\right)^2 \; if \min_{i \neq j} p_i \geq \frac{1-c}{2} \\ \left(\min_{i \neq j} p_i - c\right)\left(1 - \min_{i \neq j} p_i\right) \; otherwise \end{cases} > 0$$

iii. This implies that seller j undercuts his opponents.

iv. But in equilibrium, this strategy is NOT sustainable since seller j must conjecture that other sellers i would intentionally allow j to undercut them and net the whole market

CHARACTERIZATION 6

- The preceding argument implies that there does not exist an equilibrium—for the case all sellers have the same marginal cost—in which any given seller charges a price strictly above marginal cost c and allows other sellers to undercut him
- Hence, the only price sustainable in equilibrium is $p = c$.
- This implies that so long as there are more than 2 sellers, the price is $p = c$ and the aggregate demand is $Q = 1 - c$
- Unlike the Cournot case, this holds for each finite n and not as a limiting result.

CHARACTERIZATION 7

- Now consider the case when there are 2 sellers and $c_1 < c_2$
- There are two possible cases.
- Case 1: suppose that $c_2 \in \left(\frac{1}{2}, 1\right)$ and $c_1 \in (0, 2c_2 - 1)$, then the seller 1 may charge the monopoly price $p^m = \frac{1+c_1}{2}$ and seller 2 does not compete; otherwise, they would do so at a loss
- Case 2: If $c_1, c_2 \in (0,1)$ and $c_1 \geq 2c_2 - 1$, then the seller cannot impose the monopoly price and charges $p_1 = c_2$ and $p_2 \in [c_2, 1]$.
- Notice that if there are more than 2 sellers but $c_1 < c_2 \leq \min_{m \in \{3,4,\cdots,n\}} c_m$, then the argument does not change from the 2 seller case.

COURNOT VERSUS BERTRAND

Symmetric Case (cost is $c \in (0, 1)$)	Cournot	Bertrand	Asymmetric Case	Cournot	Bertrand
Effective Price (Price buyers pay)	$p_n = 1 - (1 - c)\left(\frac{n}{1 + n}\right)$	$p_n = c$		$p_n > c_2$	$p_n = c_2$
Quantity	$Q_n = (1 - c)\left(\frac{n}{1 + n}\right)$	$Q_n = 1 - c$		$Q_n < 1 - c_2$	$Q_n = 1 - c_2$
Limiting Behavior	$p^* = c, Q^* = 1 - c$	$p^* = c, Q^* = 1 - c$			$p^* = c_2, Q^* = 1 - c_2$