LECTURE 4

JD R-M

RECAP: LAST TIME

- Quasilinear Utility Model
  - Basic facts about monopolies

FOR TODAY

- Present first degree, monopolies. (i.e. no price discrimination)
- Common knowledge

ENVIRONMENT (BACK TO UNIT DEMAND)

- Market participants:

I. A unit mass continuum of consumers

II. A seller (1 agent)

- Actions:

I. Consumers make a purchase the good: $b^i \in \{0,1\}$

II. Seller picks the price $p \in \Re_+$

PAYOFFS

- A consumer with a valuation $\theta \sim F, \theta \in \Re_+, F$ is its CDF, observing a price of p receives payoff

$$(8)\ u(\theta, p, b) = b(\theta - p)$$

- Assume that F is a continuously differentiable function and $F'(.) \equiv f(.) \gg 0$
- Valuations are independently drawn across consumers.
- If the seller posts a price of p and a mass of $m \in [0,1]$ consumers buys the good, his profits are

$$(9)\ \pi(p, m) = (p - c)m, c \in (0,1)$$

TIMING

I. Nature draws a valuation $\theta \in \Re_+$ for each buyer and privately informs each buyer of its realization

II. Seller posts a price $p \in \Re_+$

III. Consumers, simultaneously, make a purchase decision

IV. Trade occurs and the market closes

THEORETICAL SIDE-STEP: COMMON KNOWLEDGE

- Before we continue, it is important to characterize what market participants "know".
- We will divide information into private information, public information, beliefs, and common knowledge
- Private information: These are the facts of the market in which only an agent is aware with precision
- Public Information: This is what all agents observe
- Beliefs: These are the conjectures agents have about their peer's private information.
- E.g.: In the model, only a buyer knows his valuation.

THEORETICAL SIDE-STEP: COMMON KNOWLEDGE

- SOME public information is common knowledge.
- Defining common knowledge is beyond the scope of this class, but we can intuitively present it with an example.

Example: Jorge speaks English (goodly too).

- You all know that I speak English
- But I know that you all know that I speak English
- But you know that I know that you know that I speak English
- But you know that I know that you know…so on and so forth
- A fact is common knowledge if it is known by all agents in question and there is an "understanding" among the agents that the fact is known.

COMMON KNOWLEDGE: IN THE MODEL AND WHO CARES?

- In the model: it is common knowledge

I. The timing of events

II. The actions available to each agent

III. The price posted by the seller

IV. The payoffs of each agent

V. The distribution of valuations

- This matters since relaxing what is common knowledge, completely changes how the market operates and the model's predictions.

STRATEGIES

- A monopoly strategy is a price $p \in \Re_+$ (Again, this follows from a lack of private information)
- A seller strategy is a function b: $\Re_+^2 \rightarrow \{0,1\}$ where for each price p and valuation $\theta, b(p,\theta) = 1$ states that the consumer purchases the good and $b(p,\theta) = 0$ means that he doesn't.
- I assume that a consumer who is indifferent between purchasing the good and going home will buys.
- The definition assumes that consumers behave anonymously: if two different buyers observe the same price and valuation, they make the SAME choice.
- I will ignore the market clearing condition since, in equilibrium the monopolist holds correct beliefs about his demand and supply enough of the good to meet the demand.

EQUILIBRIUM

- An equilibrium is a pair $\sigma = (p, b)$ such that

I. Given σ, the price p solves

$$(10)\ \pi(\sigma) = \max_{p \in \Re_+} (p - c) E\big(b(\theta, p)\big)$$

II. Given σ, p, θ, the choice $b(\theta, p)$ solves

$$(11)\ V(\sigma, p, \theta) = \max_{x \in \{0,1\}} x[\theta - p]$$

HOW DO WE FIND THE EQUILIBRIUM?

- Since every agents' choice is observed, we can solve this model via backwards induction.

Backwards Induction Procedure: Overview (to be applied later)

1. Go to the last stage when agents make choices and for each possible information received pick an action that maximizes the decider's payoffs
2. Next, assuming that in the preceding stage deciders follow the strategy chosen above and for each collection of information observed, pick a choice that maximizes the current decider's payoffs
3. Iterate the procedure above until one reaches the first stage when agents make choices, the resulting collection of choices parameterized by information is guaranteed to be an equilibrium.

CHARACTERIZATION

- In the last stage consumers make purchase decisions
- Assume that a consumer observes a pair(p, θ), then he decides to buy iff $\theta \geq p$
- Thus, the consumer strategy is

$$(12)\ \ b(\theta, p) = \begin{cases} 1\ if\ \theta \geq p \\ 0\ if\ \theta < p \end{cases} \equiv 1_{\theta \geq p}$$

CHARACTERIZATION: AGGREGATE DEMAND

- Given the consumer strategy, what is aggregate demand?
- Fix some price $p \geq 0$, then

$$(13)\ D(p) = E[b(\theta, p)] = E\left[1_{\theta \geq p}\right] = \Pr(\theta \geq p) = 1 - \Pr(\theta \leq p) = 1 - F(p)$$

CHARACTERIZATION: SELLER'S PROBLEM

- Seller now conjectures that given his price $p \geq 0$, aggregate demand will be $D(p)$
- He thus picks a price $p \geq 0$ to solve

$$(14)\ \pi^* = \max_{p \in \Re_+} (p - c)[1 - F(p)] = (p - c)D(p)$$

CHARACTERIZATION: SOLVING THE PROBLEM

- We can solve this problem by setting up the Lagrangian as

$$(15)\ \mathcal{L}(p,\lambda) = (p-c)[1-F(p)] + \lambda p = (p-c)D(p) + \lambda p, \lambda \geq 0$$

- The foc states

$$(16)\ \lambda + [1-F(p)] = f(p)(p-c)$$

And

$$(17)\ \lambda + D(p) = -D'(p)(p-c)$$

CHARACTERIZATION: CONTINUED

- Q: Can the price be 0?
- A: No, otherwise from (16), it holds that $\lambda + 1 = -f(0)$ or that $\lambda < 0$.
- This implies that

$$(18)\ p = \underbrace{c}_{\substack{competitive\\ price}} + \underbrace{\frac{D(p)}{D'(p)}}_{\substack{Monopoly\\ Premium}}, \hat{v}(p) \equiv p - \frac{1 - F(p)}{f(p)} = c$$

NUMERICAL EXAMPLE

- Suppose that $c = 0$ and that for each $x \in [0,1], F(x) = x^{\alpha}, \alpha > 0$
- Aggregate demand for each price $p \in \Re_+$ is then $D\ (p) = \min\{0,1 - p^{\alpha}\}$
- This implies that the seller might as well choose a price $p \in [0,1]$
- The seller's problem reduces to

$$\pi^* = \max_{p \in [0,1]} p(1 - p^{\alpha})$$

NUMERICAL EXAMPLE

- How do we solve the seller's problem?
- A: We set up the Lagrange Equation (this method holds throughout) as

$$L\ (p, \lambda_0, \lambda_1) = p(1 - p^{\alpha}) + \lambda_0 p + \lambda_1(1 - p)$$

- We now take the foc as

$$1 - p^{\alpha} + \alpha p^{\alpha} + \lambda_0 - \lambda_1 = 0$$

- We can then show that $p \in (0,1)$ as profits when $p \in \{0,1\}$ are 0. Plugging this fact to the foc above and re-organizing the equation leads us to

$$p = \left(\frac{1}{1+\alpha}\right)^{\frac{1}{\alpha}}$$

NEXT CLASS

- I will derive a general characterization to the seller's problem for a general function $F(.)$ and a marginal cost c.
- Study price discrimination and discuss the paper: Limits of Price Discrimination (AER, 2014) by Bergmann and Morris