



LECTURE 3

JD R-M

RECAP

- Studied Perfectly competitive markets when
 - Consumers have unit demand (each buyer wants 1 indivisible unit)
 - Buyers have a single good (he can sell it or keep it)
- Defined
 - Strategies and equilibrium
- We concluded that
 - Market clearing condition characterized the Equilibrium.

FOR TODAY

- Present model of perfect competition with quasilinear utility (buyers change quantity demanded)
- Defining Firms (what are they?, cost functions.)
- Why do we have monopolies?
- First-degree monopolies.



ENVIRONMENT

- Unit mass of firms and another of buyers
- Each firm j chooses its supply $c_j \in \mathcal{R}_+$
- Each buyer i chooses a demand $c_i \in \mathcal{R}_+$



FIRMS

- What is a firm? An entity that produces output and maximizes profits
- In a competitive settings, firms take the price as given
- Output is produced at a cost, described by a function $c: \mathcal{R}_+ \rightarrow \mathcal{R}_+$
- This means that to produce x it costs the firm $c(x)$
- Type of costs: fixed, variable, and sunk cost.
 - ❑ **Fixed costs:** Costs that do not change with the production level
 - ❑ **Variable costs:** Costs that vary with the level of production
 - ❑ **Sunk Costs:** Costs that cannot be re-adjusted

COST FUNCTIONS

- Assume that $c(\cdot)$ is a strictly convex,
- Common Examples:
 - i. Linear cost functions: $\forall q \geq 0, c(q) = a + bq, (a, b) \gg 0$
 - ii. Quadratic cost functions: $\forall q \geq 0, c(q) = \frac{q^2}{2\psi}, \psi > 0$
 - iii. Convex: $\forall q \geq 0, c(q) = \frac{q^{1+\psi}}{\eta(1+\psi)}, \psi, \eta > 0$

BUYER PREFERENCES

- I assume that consumers have preferences over the good in question $c \in \mathfrak{R}_+$ and a numeraire $m \in \mathfrak{R}$,
- For each $(c, m) \in \mathfrak{R}_+ \times \mathfrak{R}$, payoffs equal to

$$(1) U(c, m) = u(c) + m$$

- Assume $u(\cdot)$ is strictly increasing and strictly concave
- Assume $p_m = 1$, consumers endowment is $I > 0$, and $p_c = p$

CONSUMER PROBLEM

- A consumer problem is then

$$(2) V(p) = \max_{(c,m) \in \mathfrak{R}_+ \times \mathfrak{R}} u(c) + m \text{ s. t. } pc + m = I$$

- Since $m \in \mathfrak{R}$, then the budget constraint implies that $m = I - pc$ and the consumer problem reduces to

$$(3) V(p) = \max_{c \in \mathfrak{R}_+} u(c) - pc$$

REMARK ON THE QUASILINEAR ENVIRONMENT

- Observe that we allow negative consumption of m and that preferences are additive,
- This allows us to focus on the small market at hand, but at the cost of wealth effects,
- This framework is not useful when one is interested in general equilibrium effects,
- IF general equilibrium effects matter, then the quasilinear environment is misguided.

TIMING AND STRATEGIES

- Market participants act simultaneously and **receive no private information**
- A firm strategy is then a quantity $c_j \in \mathfrak{R}_+$ and a consumer strategy is another quantity $c_i \in \mathfrak{R}_+$,

EQUILIBRIUM

- An equilibrium is a tuple $(c_i, c_j, p) \in \mathfrak{R}_+^3$ such that
- Given p , c_j solves

$$(4) \pi(p) = \max_{c \geq 0} pc - c(c)$$

- Given p , c_i solves

$$(5) V(p) = \max_{c \geq 0} u(c) - pc$$

- Market clears: $c_i = c_j = c$

CHARACTERIZATION

- The firm problem is characterized by an f.o.c. $p - c'(c) + \lambda_0 = 0$,
- Similarly, the household problem is also characterized by the FOC $u'(c) - p + \hat{\lambda}_0 = 0$
- If $c > 0$, then

$$(6) p = u'(c) = c'(c)$$

A NUMERICAL EXERCISE

- Suppose that $\forall c \geq 0, u(c) = \ln c, c(c) = \frac{c^2}{2\psi}$, then equation (6) states that

$$(7) p = \frac{1}{c} = \frac{c}{\psi}$$

- This implies that $c(\psi) = \sqrt{\psi}, p(\psi) = \frac{1}{\sqrt{\psi}}$
- Hence, as production cost rise, i.e. $\psi \searrow 0, c(\psi) \searrow 0$ and $p(\psi) \nearrow \infty$.

QUASILINEAR VS UNIT DEMAND: A COMPARISON.

- The quasilinear model is closer to what you already learned
- Notion of equilibrium and strategy are trivial
- Illustrates that prices ensure that marginal benefits from a market equal its marginal costs

BUT

- It obfuscates the nature of trade
- It is more difficult to empirically calibrate (What is the right function $u(\cdot)$? WHEN is a market small enough)
- It often puts focus on consumers when the class prioritizes the actions of firms.

States of America



vention shall be granted under the law.

Therefore, this 5,860,492

United States Patent

Grants to the person(s) having title to this patent the right to exclude others from making, using, offering for sale, or selling the invention throughout the United States of America or importing the invention into the United States of America for the term set forth below, subject to the payment of maintenance fees as provided by law.

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MONOPOLIES

- What are monopolies?
- Why are they legal?

HOW DO WE GET MONOPOLIES?

- Natural Monopolies
- Patents, Copyright, trademarks
- Barriers to entry



NATURAL MONOPOLIES

- Def: An industry is said to exhibit a natural monopoly provided that its cost function is sub-additive: $\forall x, y \in \mathfrak{R}_+, c(x + y) \leq c(x) + c(y)$.
- Example: High fixed costs,
- Examples of natural monopolies: Utility companies; National;
*Professional Sport Leagues; railroads; *telecommunications





ARE NATURAL MONOPOLIES ALWAYS MONOPOLIES?

- A: No. Here are a few examples...
- US telephone service provides
- Schengen level railroad service
- NY Subway system prior to June 1940

PATENTS, COPYRIGHTS, TRADEMARKS, INDUSTRY SECRET

- National governments provide non-competition
- In Article 1, Section 8 of the U.S. Constitution (1789), it states that Congress has the power to enact patent protections

“To promote the progress of science and useful arts, by securing for limited times to authors and inventors the exclusive right to their respective writings and discoveries”.



BARRIERS TO ENTRY AND NON-COMPETITIVE PRACTICES

- Firms also tend to partake in non-competitive practices. Here are some examples
 - I. Selling goods below cost: e.g. <https://www.cnbc.com/2016/05/19/wal-marts-low-price-commitment-should-send-shivers-across-retail.html>
 - II. Cartel Behavior (multiple firms choose not to directly compete with each other): e.g. OPEC, HUD.
 - III. Government protection: e.g. Nationalized industries.

For more information on similar topics, look up James A. Schmitz, Jr., <https://www.minneapolisfed.org/people/james-a-schmitz-jr>

ANTI-TRUST LEGISLATION

- Pertinent US Laws: Sherman Act (1890), Clayton Act (1914), Federal Trade Commission Act (1914)
- Enforcement: FTC and US Department of Justice
- Main objectives: promote competition AND efficiency
- There is major disagreement in this front (e.g. some want to protect "small firms" from their large counterparts).



ENVIRONMENT (BACK TO UNIT DEMAND)

- Market participants:
 - I. A unit mass continuum of buyers
 - II. A seller (JUST 1 SELLER)
 - Actions:
 - I. Each buyer i , chooses a quantity to buy $a_i \in \{0,1\}$
 - II. Seller picks the price $p \in \mathfrak{R}_+$
- Note: since the seller is the unique seller, he PICKS the price. There is no more invisible hand.

PAYOFFS

- A consumer with a valuation $\theta \sim F, \theta \in \mathfrak{R}_+, F$ is its CDF, observing a price of p receives payoff

$$(8) v(\theta, p) = a_i(\theta - p)$$

- Valuations are independently drawn across consumers.
- If the seller posts a price of p and a mass of $m \in [0,1]$ consumers buys the good, his profits are

$$(9) \pi(p, m) = (p - c)m, c > 0$$

TIMING

- Market Operates as follows
 - I. Nature draws a valuation $\theta \in \mathfrak{R}_+$ for each buyer and privately informs each buyer of its realization
 - II. Seller posts a price $p \in \mathfrak{R}_+$
 - III. Consumers, simultaneously, make a purchase decision
 - IV. Market closes.

STRATEGIES

- A monopoly strategy is a price $p \in \mathbb{R}_+$ **(He receives no private information)**
- A buyer strategy is a function $b: \mathbb{R}_+^2 \rightarrow \{0,1\}$ where for each price (p, θ) , $b(p, \theta)$ is quantity purchased
- I assume that a consumer who is indifferent between purchasing the good and going home will buy.
- The definition assumes that buyers behave anonymously **(Any two buyers with the same information will make the same action)**
- I will ignore the market clearing condition since, in equilibrium the monopolist holds correct beliefs about his demand and supply enough of the good to meet the demand.

EQUILIBRIUM

- An equilibrium is a pair $\sigma = (p, b)$ such that
 - I. Given σ , the price p solves

$$(10) \pi(\sigma) = \max_{p \in \mathbb{R}_+} (p - c)E(b(\theta, p))$$

- II. Given σ, p, θ , the choice $b(\theta, p)$ solves

$$(11) V(\sigma, p, \theta) = \max_{x \in \{0,1\}} x[\theta - p]$$