# LECTURE 6

JD R-M

# FOR TODAY

- Auctions!

I. What are auctions, why study them.

II. History of Auctions

III. Game Theory and Mechanism Design

IV. Environment at hand

V. Standard Auctions

# WHY STUDY AUCTIONS ?

- 3 reasons:

I. They are common markets

II. Small markets with clear asymmetric information: allows us to study price formation

III. It is the most successful (theoretical and empirical) field in all social science (and most science in general [with the potential exception of quantum mechanics] )

# WHAT ARE AUCTIONS FOR *US*?

- Monopolies with a single, indivisible item and finitely many buyers
- Now each buyer's choice affects the market
- We allow the monopolist to pick how he interacts with buyers
- This will require us thinking of game theory AND of reverse engineering games
- We need game theory anyways in order to think of competition later on…

# WHERE DOES ONE SEE AUCTIONS?

- Art  sales(e.g. Sotheby's, Christie's)
- Wine sales and rare items
- E-bay (most common nowadays)
- Government sale of contracts (procurement), access to natural resources, radio frequencies, utility monopolies, bonds
- Estate Sales
- In general: Sale of items with a. very limited, fixed supply and (often) few potential buyers known.

# AUCTIONS ARE INSIGHTFUL

- Since these markets are small, we can directly model how agents:

I. Trade

II. Learn from each other

III. Compete

IV. Seller chooses the way agents interact and ”formalizes his market”

V. Formally study information rents and cost of screening.

# NOBEL LAUREATES IN AUCTION RELATED FIELDS

- List of economist (contribution)

I. William Vickrey (What's the socially optimal auction? Principle of Revenue Equivalence)

II. Leonid Hurwicz (Fathered the broader field of Mechanism Design (1 of our 9 Econ Nobel Laureates))

III. Roger Myerson (Profit maximizing auction for a single item, the revelation principle (we will discuss his paper in class))

IV. Eric Maskin (Generalized Myerson and answered how to get things done in the real world)

V. Paul Milgrom (Theory and implementation of optimal multi-good auctions [I'm a huge fan of this guy])

VI. Robert B. Wilson (Auction theory and connection to IO [your first 3 lectures were inspired by his work])

# HISTORY OF AUCTIONS

- 500 B.C.: According to Herodotus, Babylonians  auctioned off young women for marriage (source below) and used the funds to pay dowries for the poorest families in the village. Please be aware that this is Herodotus: the father of history and citing without checking his sources OR critical thinking.
- 200 B.C.-470 A.C.E.: Roman Legionnaires auctioned off the spoils of war that they plundered in campaign
- 193 A.C.E: When the Praetorian guards killed Roman Emperor Pertinax, they auctioned off the Roman Empire in an English auction. The winning bid was 25,000 sesterces per guard by Didius Julianus. He was beheaded two days later---now THAT'S a winner's curse (Krishna 2002).
- 1634-1637: Tulip Bulb auctions were conducted in the Netherlands. Bubonic Plague led to fewer bidders and hence the collapse of tulip prices (Garber 2000)
- 1744: Sotheby's is founded
- 1766: Christie's is founded
- 1929: US Treasury began auctioning Gov't bonds
- 1993: Bill Clinton sanctions the Spectrum auctions

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# BUT FIRST…LET ME DO SOME GAME THEORY

- In order to model *how* buyers and sellers interact, we must model it carefully
- The area of economics that explicitly models strategic interactions between agents when the outcome crucially depends on each agent is called "Game Theory"
- But what *is* a game?

Definition: A normal form game $\Gamma$ consists of

I. A set of agents $I$ (This denotes *who* is involved)

II. For each agent $i \in I$, there exists a non-empty set of actions $A_i$ (what can agent $i$ do)

III. A set of outcomes $O$ (what can happen)

IV. Mapping from collections of actions to outcomes $g: A = \prod_{i \in I} A_i = \{(a_i)_{i \in I} | \forall i, a_i \in A_i\} \rightarrow O$ (it tells us: given a collection of actions taken by each player, what outcome occurs?)

V. For each agent $i \in I$, there exist a payoff function $v_i: O \rightarrow \Re$ or one defines $u_i: A \rightarrow \Re, \forall a \in A, u_i(a) = v_i\big(g(a)\big)$.

# EXAMPLES OF GAMES (WILL DISCUSS IN CLASS)

- Some examples you might have seen:

I. The competitive market game
II. The monopolist markets
III. Prisoner's Dilemma
IV. Battle of the Sexes (i.e. Beyonce or Skrillex [if you learned it from me])
V. Tic, Tac, Toe
VI. Chess
VII. Auctions
VIII. Voting

# HEURISTIC EXPLANATION

- A normal form game answers explicitly:

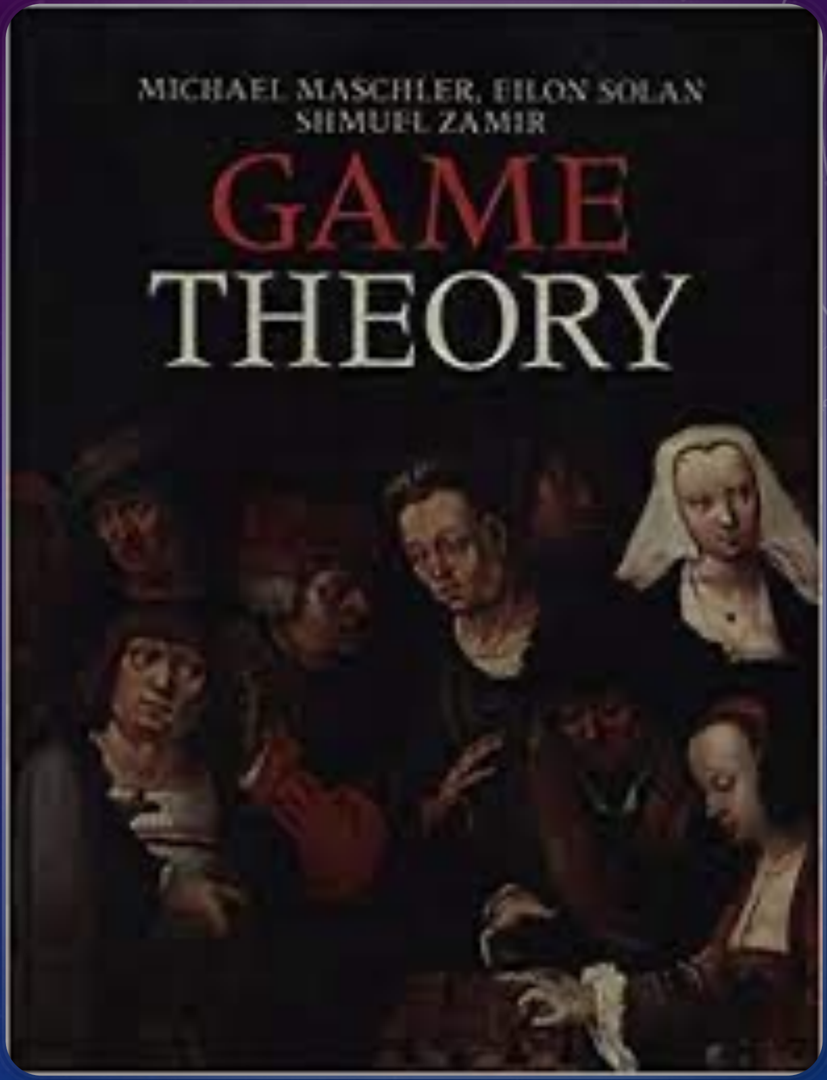
I. Who is involved?

II. What individuals do?

III. What are the possible outcomes?

IV. What are the outcomes of collective choices?

V. How do individuals value outcomes?



# BAYESIAN GAMES

- We will, in general, concern ourselves with games where payoffs are not perfectly understood by everyone
- We put in “valuations” (or formally called types)

Definition: A Bayesian Game $\Gamma_b$ consists of

I. A set of agents $I$,

II. Set of Outcomes $O$,

III. For each agent $i$, there exists non-empty sets of actions and types: $A_i, \Theta_i$ , respectively

IV. For each agent $i$, there exists a distribution $\pi_i$ denoting odds of

V. A function $g: A \to O$,

VI. For each agent $i$, there exists functions $v_i: \Theta_i \times O \to \Re$ or $u_i: \Theta_i \times A \to \Re, \forall(\theta_i, a), u_i(\theta_i, a) = v_i\big(\theta_i, g(a)\big)$

# HEURISTIC EXPLANATION

- A Bayesian game is a normal form game where it is not common knowledge what are individual preferences
- It is useful to model interactions where there exists uncertainty regarding the game played.
- Will be and has been indispensable when seriously thinking about markets.

# BACK TO COMMON KNOWLEDGE

- What's common knowledge in a normal form game?
- A: Everything $\Gamma = \big(I, O, g, (A_i, v_i)\big)$.
- What's common knowledge in a Bayesian Game?
- A: the structure of the game $\Gamma_B = \big(I, O, g, (A_i, v_i, \Theta_i, \pi_i)\big)$
- What's private information for some agent $i$ in a game?
- A: Realization of $\theta_i$.

# MECHANISM DESIGN

- Suppose we have a

I. A set of players $I$ and a ”designer” (social planner, seller, auctioneer, et cetera)

II. A set of outcomes $O$

III. For each agent in i., $(\Theta_i, \pi_i, v_i)$

- Designer would like to pick a Bayesian Game and an equilibrium in such game $\Gamma_b = \left(I, O, g, (v_i, A_i, \Theta_i, \pi_i)\right)$ such that

I. Agents $i$ want to play the game (if not payoff received is 0)

II. The outcome of the game gets him his favorite outcome among possible equilibria.

- This is reverse engineering a game to get an outcome others are willing to follow along
- It will allow us to “design markets”, better yet see why markets function as they do.

# ENVIRONMENT ("PRIMITIVES")

- The most studied Mechanism Design environment is an auction, let us define the environment (or primitives)
- The environment I will consider in class consists of:

I. A seller and $n \geq 1$ buyers (i.e. a set of players is $I = \{1,2,\cdots,n\}$)

II. The possible outcomes are: (a) who gets the good (the seller or one of the buyers) (b) how much does each play $(p_i)$

III. For each $i, \Theta_i \subset \Re_+$ are the valuations, $\pi_i$ distribution over valuations (think of $F$), and payoffs equals is

$$u_i(o, \theta_i) = \theta_i 1_{I\ won} - p_i$$

- Seller's problem is finding the auction that maximizes his profits
- It turns out that can English Auction do the trick (Proven by Myerson (1981)).

# BUT WHAT KINDS OF AUCTIONS EXIST?

- English Auction: Bidding starts off at some minimal level and buyers submit increasingly higher bids until no new bids arrive and the highest bid wins the good and pays his bid*

Examples: Christies's, Sotheby's, Storage Wars

- Dutch Auctions: Bidding starts at a high value and gradually falls until the first buyer decides to bid and wins the good.

Examples: Tulips, IPOs

- First price auction: Bidders privately and simultaneously submit bids above a given threshold, the highest bid wins a wins the good and pays his bid
- Second price auction: Bidders bid as above, the highest bid wins BUT pays the second highest bid*
- All pay auctions: All bidders submit a bid and pay it, one bidder wins the good (e.g. R&D)

# NEXT CLASS

- I will model some important auctions
- Revenue Equivalence
- But What's the profit maximizing auction anyways?
- How does one even go about proving these things?