LECTURE 12

JD R-M

FOR TODAY

- Q: Does the order of play matter?
- A: Yes.
- Plan for today:

1. Review Oligopoly (Symmetric Producers)
2. Stackelberg model

ENVIRONMENT

- Agents: $n \geq 2$ Producers and a unit mass continuum of Consumers
- Actions:

i. Producers: picks quantity to produce $q_j \in [0,1]$

ii. Consumers: purchase $x \in \{0,1\}$ units of the good

- Payoffs:

i. A consumer buying x units at a price of p nets a payoff of $x[\theta - p]$

ii. A producer producing q units at a price of p gets profits of $q(p-c), c \in (0,1)$

- Technical points: All values $\theta \sim U[0,1]$ are drawn pairwise independent of each other.

COURNOT TIMING

1. Nature draws valuations and privately informs each buyer of their own realization
2. Producers announce their supply
3. Consumers make purchase decision
4. Market closes

STRATEGIES

- A consumer strategy is a function $b: [0,1]^{n+1} \rightarrow \{0,1\}$ denoting number of units bought as a function of quantities chosen and valuations
- A producer $j's$ strategy is a quantity $q_j \in [0,1]$

EQUILIBRIUM

- An equilibrium is a tuple $\sigma = \left(b, \left(q_j\right)_{j=1}^{n}, p\right)$ such that

i. For each $\theta, \left(\tilde{q}_j\right)_{j=1}^{n}, p$, $b\left(\theta, \left(\tilde{q}_j\right)_{j=1}^{n}\right)$ solves $CS\left(\theta, \left(\tilde{q}_j\right)_{j=1}^{n}, p\right) = \max_{x \in \{0,1\}} x(\theta - p)$

ii. Given beliefs $q_{-j}, b(.)$, each producer j picks q_j solving $\pi_j\left(q_{-j}, b\right) = \max_{q_j \in [0,1]} q_j\left(p\left(b, q_{-j}, q_j\right) - c\right)$

iii. Beliefs are consistent: $p = p\left(b, \left(\tilde{q}_j\right)_{j=1}^{n}\right)$

iv. Markets clear: $E[b(\theta, q_1, q_2)] = \sum_{j=1}^{n} q_j = Q_C$

CHARACTERIZATION 1

- As before $\forall \left(\tilde{q}_j\right)_{j=1}^{n}, \theta, b\left(\theta, \left(\tilde{q}_j\right)_{j=1}^{n}\right) = 1_{\theta \geq p}$
- Demand is then $D(p) = 1 - p$
- From market clearing and consistency: $1 - p = \sum_{j=1}^{n} q_j \leftrightarrow p\left(\left(q_j\right)_{j=1}^{n}\right) = 1 - \sum_{j=1}^{n} q_j$

CHARACTERIZATION 2

- A producer's problem, given equilibrium beliefs reduces to solving

$$(1)\ \pi_j(q_{-j}) = \max_{q_j \in [0,1]} q_j\left[1 - \left(q_j + Q_{-j} + c\right)\right]$$

- For $Q_{-j} = \sum_{i \neq j} q_i$
- Solution is given by setting up the full Lagrange equation as before and deriving the foc and slackness conditions
- I conjecture that both pick $q_j \in (0,1)$, so the foc reduces to

$$(2)\ 1 - (c + Q_C) = q_j$$

CHARACTERIZATION 3

- In equilibrium, each buyer picks the same quantity q, and $Q_c = nq$, so the foc implies that

$$(4)\ \forall j, \qquad q_j = q = \frac{1-c}{n+1}$$

- Plugging this into the profit functions, it holds that

$$(5)\ \forall j \in \{1,2\}, \qquad \pi_j^C = \pi^C = \left(\frac{1-c}{n+1}\right)^2 > 0$$

IMPLICATIONS

- When producers face the same per unit cost (or cost function) and compete in quantities

i. The produce the same level of output

ii. They generate the same level of profit

- A basic issue with this model is that producers act simultaneously
- Q: If one can make a decision before his peer can he gain from such decision?
- A: Yes.

ENVIRONMENT AND TIMING 2 SELLER EXAMPLE:

- Fix the environment as before
- Timing is now the following:

1. Nature draws valuations and privately informs each buyer of their own realization
2. Producer 1 picks his supply
3. Producer 2 picks his supply
4. Consumers make purchase decision
5. Market closes

STACKELBERG STRATEGIES

- A consumer strategy is a function $b: [0,1]^3 \rightarrow \{0,1\}$ denoting number of units bought as a function of quantities chosen and valuations
- A producer 1 strategy is a quantity $q_1 \in [0,1]$
- Producer 2 strategy is a function $q_2: [0,1] \rightarrow [0,1]$ such that for each $q \in [0,1]$, $q_2(q) \in [0,1-q]$

EQUILIBRIUM STACKELBERG

- An equilibrium is a tuple $\sigma = (b, q_1, q_2, p)$ such that

i. For each $\theta, \tilde{q}_1, \tilde{q}_2, p, b(\theta, \tilde{q}_1, \tilde{q}_2)$ solves $CS(\theta, \tilde{q}_1, \tilde{q}_2) = \max_{x \in \{0,1\}} x(\theta - p)$

ii. Given $q_1, and\ beleifs\ about\ b(.)$, producer 2 picks q_2 solving $\pi_2(q_1, b) = \max_{q_j \in [0,1-q_1]} q_j(p(b, q_1, q_2) - c)$

iii. Given beliefs regarding, $b(.), q_2(.)$, producer 1 picks q_1 solving $\pi_1(q_2, b_2) = \max_{q_1 \in [0,1\ ]} q_2\left[p\left(b, q_1, q_2(q_1)\right) - c\right]$

iv. Beliefs are consistent: $p = p(b, q_1, q_2)$

v. Markets clear: $E[b(\theta, q_1, q_2)] = q_1 + q_2 = Q_s$

CHARACTERIZATION 1

- As before $\forall q_1, q_2, \theta, b(\theta, q_1, q_2) = 1_{\theta \geq p}$
- Demand is then $D(p) = 1 - p$
- From market clearing and consistency: $1 - p = q_1 + q_2 \leftrightarrow p(q_1, q_2) = 1 - (q_1 + q_2)$

CHARACTERIZATION 2

- Producer 2 rightly conjectures demand and observes $q_1 \in [0,1]$
- He solves

$$(6)\ \pi_2(q_1) = \max_{q_2 \in [0,1-q_1]} q_2[1-(q_1+q_2)-c]$$

- Setting up the right Lagrange yields an foc and slackness condition

$$(7)\ [1-(c+Q_c)] = q_2$$

CHARACTERIZATION 3

- This implies that Producer 2's supply as a function of producer 1's supply equals to

$$(8)\ \forall q_1 \in [0,1], q_2(q_1) = \left[\frac{1-(c+q_1)}{2}\right]^+$$

CHARACTERIZATION 4

- Producer 1 now conjectures the right buyer and producer 2 behavior
- He then solves

$$(9)\ \pi_1(b, q_2) = \max\left\{\max_{q_1 \in [0,1-c]} q_1\left[1 - c\ - \frac{1 - c - q_1}{2} - q_1\right], \max_{q_1 \in [1-c,1]} q_1(1 - c\ - q_1)\right\}$$

- This is equivalent to solving

$$(10\ )\ \pi_1(b, q_2) = \max_{q_1 \in [0,1-c]} \frac{q_1(1 - c - q_1)}{2}$$

CHARACTERIZATION 5

- Setting up another Lagrange equation leads to a $q_1 \in (0,1)$ solving

$$(11)\ q_1 = \frac{1-c}{2}$$

- Plugging this into $q_2(.)$, leads to a producer 2 output of

$$(12)\ q_2 = q_2\left(\frac{1-c}{2}\right) = \frac{1-c}{4}$$

CHARACTERIZATION 6

- Total output is then

$$(13)\ Q_S = q_1 + q_2 = \left(\frac{3}{4}\right)(1 - c) > Q_C$$

- Price equals to

$$(14)\ P_S = 1 - Q_S = \frac{1 + 3c}{4} < P_c$$

CHARACTERIZATION 7

- Produce 2's profits equal to

$$(15)\ \pi_2^S = \left(\frac{1-c}{4}\right)^2 < \pi^S$$

- Producer 1's profits equal to

$$(16)\ \pi_1^S = \frac{(1-c)^2}{8} > \pi_1^S$$

CONCLUSIONS FROM 2 PRODUCER CASE

- The Stackelberg market relative to the Cournot market

i. Has a LOWER equilibrium price

ii. Has a HIGHER level of equilibrium output

iii. The leader nets strictly higher profits than in the Cournot market

iv. The follower nets strictly lower profits than in the Cournot market

- Punchline 1: it is profitable to be the leader and not the follower
- Punchline 2: Inequality among producers can be a blessing
- Caveat: If the leader is sufficiently more productive than the follower, he can behave as a monopolist.

EXAMPLE WITH N SELLERS, C=0

- We can define a market where there are n seller and costs are normalized to 0
- Normalizing the per unit to 0 simplifies that outcome and avoids some technical issues.
- Timing:

i. Nature draws types and privately informs each buyer of their type

ii. Seller 1 picks his quantity and announces it

iii. Seller 2 then picks his quantity and announces it

iv. The others seller follow suit in order

v. The consumers make their purchase decision

CHARACTERIZING THE EQUILIBRIUM

- The consumer choice remains identical to what it has been through most of the class
- Now, the last seller observes a demand $Q_{-n} = \sum_{j=1}^{n-1} q_j$ and solves

$$(17)\ \pi_n(Q_n) = \max_{q_n \in [0, Q_n]} q_n(1 - Q_{-n} - q_n)$$

- His optimal choice then solves the foc

$$(18)\ q_n = \frac{(1 - Q_{-n})}{2}$$

JTH PRODUCER'S PROBLEM

- The jth $, 1 \leq j < n$ producer then observes a quantity $Q_{-j} = \sum_{i=1}^{j-1} q_i$ and conjectures that subsequent producers pick a quantities such that $Q_{+j}(Q_{-j}, q_j) = \alpha_j(1 - Q_{-j} - q_j)$ for some $\alpha_j \in (0,1)$ and solves

$$(19)\ \pi_j(Q_{-j}) = \max_{q_j \in [0, Q_{-j}]} q_j[1 - Q_{-j} - q_j - \alpha_j(1 - Q_{-j} - q_j)]$$

- The foc characterization from above implies that $q_j(Q_{-j}) = \frac{1-Q_{-j}}{2}$
- Notice that at the initial value $Q_{-1} = 0$ and hence: for each producer j $q_j = 2^{-j}$

PRICE, QUANTITIES, AND PROFITS

- The Price then satisfies

$$(20)\ p = 1 - \sum_{j=1}^{n} q_j = 1 - \sum_{j=1}^{n} \left(\frac{1}{2}\right)^j = 1 - \left(\frac{1}{2}\right)\left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right) = \left(\frac{1}{2}\right)^n$$

- Quantity equals to

$$(21)\ Q_c = 1 - \left(\frac{1}{2}\right)^n$$

- Profits then satisfy

$$(22)\ \forall j, \pi_j = \frac{1}{2^{n+j}}$$

PUNCHLINE FROM N PRODUCER CASE

- When there are many producers, those who make their choices first net a higher payoff than those who follow them
- This model predicts an exponential distribution of market shares
- Nonetheless, prices and profits still fall to the marginal cost as the number of producers increases